Solid Abelian Groups

Idea: Want some kind of "completeness" condition on Cond A5

Why? Peter explained that

\[ \text{Hom}(A, B), \]

\[ \text{RHom}(A, B) \]

in Cond A5 are "good".

Tensor product \(- \otimes -\) on Cond A5 is defined as:

\[ \mathcal{O} [S] \otimes \mathcal{O} [T] = \mathcal{O} [S \times T] \]

let \[ \mathcal{O} \mathcal{P} \otimes \mathcal{P}, \mathcal{P} \otimes \mathcal{P} \]
In fact, $(\mathcal{P} \otimes \mathcal{P})(\mathbb{Z}) = \mathcal{P} \otimes \mathcal{P}$ (alg. dual tensor product).

We'd rather have

\[
\mathcal{P} \otimes \mathbb{Z} = \mathbb{Z}
\]

\[
\mathcal{P} \otimes \mathbb{Z}_2 = 0
\]

**Solid Ab ≤ Cond Ab**

closed under limit & colimit, E left adjoint to \( \operatorname{Hom} \)

completed \( \hat{\mathcal{P}} \) will be

\[
\hat{\mathcal{P}} \otimes \hat{\mathcal{N}} = (\mathcal{P} \otimes \mathcal{N})^*.
\]

In fact, \( \text{Solid Ab} \) will be an abelian category.
Abstract framework:

Let $A$ be an abelian category generated by compact projectives, and suppose given a functor $A^\mathbb{P} \xrightarrow{L} A$.

With respect to $\text{id} \rightarrow L$.

If $M^{\mathbb{P}}$ is a colocal, then $A \text{L}(M_i) \rightarrow \text{L}(N_i)$.
then
\[
\text{RHom}(C, M) \cong \text{RHom}(C/L, M)
\]
\[
\forall C \in \mathcal{A}^{cp}
\]

Then: 1) The following full subcategories of $\mathcal{A}$ agree:

\[
\begin{align*}
&\text{EMEA, a cokernel as above} \\
&\text{EMEA, a cokernel as above} \\
&\text{EMEA, an } \xi_0 \text{ as above} \\
\end{align*}
\]

& form abelian subcategories closed under all limits, colimits, extensions.

2) $L$ extends uniquely to a colim-preserving functor

\[
L : \mathcal{A} \to \mathcal{A}^{cp}
\]
\[ A = 0 \left( 1 + \left( \frac{t}{n} \right) \right) \]

\[ \text{for } n \geq 1, \text{ and } t \geq 0. \]

The result is also true for the case where the content is not properly scanned or transcribed.
Proof: Suppose \( \gamma \) and \( \lambda \).

\[ \otimes C_i \rightarrow M \]

\[ \forall i \in I \]

\[ \otimes C_i \in \text{satisfies } Y \]

\[ \otimes \text{CD}_i \rightarrow M \] \( \nu \in \text{satisfies } Y \)

---

Remark: If \( A \) has a compact w/ colours in each variable, and if have the candidate \( \gamma \) not just for \( A \) hom but for \( A \)hom, then we get a unique tensor product on.
\( L(\lambda) \leq A \)
making \( A \cong L(\lambda) \)
symmetric monoidal.

\( L(\lambda) \)

\[ (\star) \]

\( \Rightarrow \) same statement, but with kernel instead of cokernel

\( \Rightarrow \) same statement, but with complex instead of \( \mathbb{C} \)-modules

Some new facts from cret proj and csgps to \( \text{Con} \geq A_b \).

\( \mathcal{M} \rightarrow \mathcal{C}(5), \) Inc.
Def: If $S$ is each disk, product set, set

$$\mathbb{C}[S] = \lim_{S_i} \mathbb{C}[S_i]$$

$S_i \rightarrow S$ if finite set

Note: $\mathbb{C}[S] \rightarrow \mathbb{C}[S_0]$.

We need to check $R\text{Hom}(\mathbb{C}[S], R)$.

Thm: (Specker) $\mathbb{C}(S, \mathbb{R}) = \bigoplus_{i} \mathbb{R}$

a tree at $S$.
Most basic case of $\mathfrak{a}$:

\[ \Theta(\mathfrak{a})/(\mathfrak{a}) = \text{RHom}(\mathbb{Z}[\mathfrak{a}], \mathbb{Z}) \]

Reflex: \[ \text{RHom}(\mathbb{Z}, \mathbb{Z}) = \mathbb{Z} \]

\[ \Theta(\mathfrak{a})/(\mathfrak{a}) = \text{RHom}(\mathbb{Z}[\mathfrak{a}], \mathbb{Z}) \]

\[ \approx \text{RHom}(\mathbb{Z}[\mathfrak{a}], \mathbb{Z}) \]

\[ \approx \mathbb{Z}[\mathfrak{a}] \]

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Recall: Even-Belju resolution \[ \text{any compact as qp41b resolved} \]
But again,

Rethink (The orbital)

So only need to rethink

 concatenate with this.

Pseudo-coherent

Ex. A = - [ ]

A is a Hilbert space.

In any projective, y = 06 (CA)

can be assessed.

In terms of (CA),

Rethink (The orbital)
(\mathbb{R} \to \mathbb{R} \to \mathbb{R}/\mathbb{Z} \\
\Rightarrow \mathbb{R} \text{ pseudo-abelian})

\text{Cok} \text{Lim} \quad \mathbf{RHom}(\mathbb{R}, \mathbf{R})

= \bigoplus \mathbf{RHom}(\pi_1, H_1)

= \bigoplus \mathbb{Z} \oplus \mathbb{Z}

The general case is more complicated.

How to calculate -\otimes -?

Presumably this is, then

\text{coker}(\text{lim}(T_i))

\mathbb{T} \otimes \mathbb{Z} \\
\text{I} \otimes \mathbb{Z}

I \times \mathbb{Z}
(infinite distributivity)

\[ \exists \pi : \mathcal{P} \otimes \mathcal{P} = \mathcal{P} \]

\[ \pi \otimes \pi \rightarrow \pi \sim 0 \]

In general, if \( A \subset B \subset \text{profinite sgp.} \)

\( A \otimes B \) cannot be filtered wrt. \( \mathcal{P} \)

\[ \pi \otimes \pi = 0 \ 	ext{iff} \]

\[ \text{T} \otimes \text{T} = \text{T} \]

\[ \mathcal{P} \otimes \mathcal{P} = (\mathcal{P} \otimes \mathcal{P}) \text{[T]} \]

And

\[ T \mathcal{P} \quad \text{for} \quad \mathcal{P} \]
\text{V} \ominus \text{W} = \text{W} \ominus \text{V}

(1) In practice, \(-\ominus\) does work like a completed tensor product.

\text{Rk}: \quad 12 \circlearrowright = 0

\text{If Solid}_{\text{Acc}}\text{t} \rightarrow \text{Solid}_{\text{Acc}}\text{t}\text{h} \quad \text{Mord}: \text{Solid}_{\text{Acc}}\text{t}\text{h}

12 \text{ a non-archimedean kind of completive.}

\text{Completeness} \quad \text{w.r.t. linear topology}
Why is solvability solid like complete?

\[ \text{Hom}(\mathbb{C}[S, \mathcal{E}], \mathbb{S}) \]

"Measures on \( S \)"

Why isn't it not like completeness?

1) It's only complete "in the compacts" (\( \mathcal{C} \) quasi-completeness)

2) Mathinker: "No connection to Hausdorff"
\( 0 \to \mathcal{O}_S \to \mathcal{O}_X \to \mathcal{O}_T \to 0 \)

\[ M_{\mathcal{O}} \otimes N_{\mathcal{O}} = (M \otimes N)_{\mathcal{O}} \]

**Proof:** By restriction,

\[ \mathcal{I}(S) = \frac{\lim_{i \to \infty} \mathcal{O}_C(S_i)}{S} \]

for \( S \) exact, discrete, profinite.

**What about more general \( S \)?**

**Remark:** \( M \) pseudo-coherent

\( S \) solid ab. gp.

\[ M \cong \text{RHom}(\text{RHom}(M, \mathcal{O}_S), \mathcal{O}_T) \]
\[ \mathbf{M} = \mathcal{C} \mathcal{S}^* \cdot \mathbf{L} \]

\[ \mathcal{C} \mathcal{S}^* \cdot \mathbf{L} \quad \text{is compact from space} \]

\[ \overset{\mathcal{R} \text{Ham}}{\mathcal{R} \text{Hom}} (\mathcal{R} \text{Hom}(\mathcal{C} \mathcal{S}^* \cdot \mathbf{L}) \quad \text{in} \quad \mathbb{R}^2) \]

\[ \overset{\mathcal{R} \text{Ham}}{\mathcal{R} \text{Hom}} (\mathcal{R} \text{Hom}(\mathcal{T} (S^2, \mathbb{R}) \quad \text{in} \quad \mathbb{R}) \]

\[ \overset{\text{Ex: S product}}{\lim_{S_i}} \quad \mathbf{S}^2 \]

\[ \overset{\text{Ex: S fibre}}{\lim_{S_i}} \quad \mathcal{L} (S^2 \mathbf{C}) \]

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\[ \overset{\text{Ex: S fibre}}{\lim_{S_i}} \quad \mathcal{L} (S^2 \mathbf{C}) \]
\[ H \times D e n ( \mathcal{C} ) \]

\[ \mathcal{D}(S) : \text{Homology of } S \]

Solidification commutes with cells.

\[ \Rightarrow \text{Same statement true for arbitrary } \mathcal{C} \text{ & } \mathcal{D} \]

Synthesis approach to Solidify:

\[ A^g \preceq A \]

\[ \text{coact proj generators} \]

\[ \Rightarrow \]

\[ \Theta_j \rightarrow \Theta C_i \rightarrow M \]

\[ i \]

\[ \Theta_j' \rightarrow \Theta C_i' \rightarrow N \]

\[ \text{Complete \& determined by maps between coact proj.} \]
If $A^p$ is small,

$$A = \text{Fun} \left( \tilde{A}^p, \text{Ab} \right)$$

$\omega \rightarrow (c + c) \text{Hom}(\mathcal{C}_1 \text{M})$

$\text{Solid} \_k$ (k-codensed)

Each map $\mathbb{T}_{n-1} \mathbb{T}_{n}$

$\exists$ dec of $E_{\mathcal{C}} - E_{\mathcal{D}}$

$\text{Solid} \_k = \text{Fun} \_k (\text{FreeAb}_k, \text{Ab})$

Could forget that it sits in $\text{CondAb}$

& just work with this.

$G \rightarrow \text{SolidAb gp}$
Whitehead's problem

Is the following true:

\[ A \in Ab, \quad \text{Ext}^1(A, \mathbb{Z}) = 0 \]

\[ A = \mathbb{Z} \oplus ? \]

Answer (Shelah): It depends!

(on axioms beyond ZFC)

Thus: If \( A \in Ab \), then

\[ \text{Ext}^1(A, \mathbb{Z}) = 0 \]

\[ \Rightarrow A = \mathbb{Z} \oplus ? \]

Note: \( \text{Ext}^1(A, \mathbb{Z}) \) is still undefined

If underlying ab gp
Theorem: The sum of the angles of a triangle is 180 degrees.

Proof: Consider a triangle ABC. Draw a line parallel to BC through A, meeting AC at D. Then, since AD is parallel to BC, we have

\[ \angle A + \angle BAC + \angle B = 180^\circ. \]

Thus, the sum of the angles of a triangle is 180 degrees.
1) 1.2 \ \text{cm}^2 \\
\text{cap}

\text{color} \left( \frac{1}{42} - \frac{\pi}{2} \right)

\text{rock inc.}

2) Is it flat?

\text{i.e. } M \in \text{Solid}\text{2}

\begin{align*}
\frac{1}{2} \otimes M &= \frac{\pi}{2} \otimes k?
\end{align*}

\text{Solid}\text{2} = \frac{1}{2} \otimes \text{Solid}\text{1} - \text{Solid}\text{1} \otimes \text{Solid}\text{2}

\text{Solid}\text{2} = \rho \otimes \tau

\rho \otimes \tau = \rho \otimes \tau

\text{Solid}\text{2} = \rho \otimes \tau
solid \( Z \) is an \( \mathfrak{S} \) strict enough quasiprecategory

\[
\text{(PRE)} / \mathfrak{S}p = \text{PRE}
\]

Here \( 1 \) \& \( 2 \) can be assumed.

1) \( \text{qct objects } = \text{colunels } (\text{PRE} + \text{PRE}) \)

are exactly the \( \text{pre} \) proper abelian \( \text{qps} \)

2) \( \text{PREp} \) is flat \( \mathfrak{S} \)

\( 1 = \) solid \( \mathfrak{S} \) : Ind (\( \text{pro}_{\mathfrak{S}} \text{-ab qps} \))

= \( \text{Ind} (\text{Pro}_{\mathfrak{S}} (\text{finite qps})) \)

How do we prove 2)?
\[ \mathbb{T} \mathbb{P} \otimes \mathbb{M} \text{ true in deg 0!} \]

But then \( \mathbb{M} \otimes \mathbb{T} \mathbb{P} = \mathbb{T} \mathbb{M} \)

More general version of 21:

If \( \mathbb{M} \) is qsep & p-torsion free, then \( \mathbb{M} \) is flat.

E.g., \( \mathbb{M} \) qsep \( \mathbb{Q}_p \)-module

\[ \underline{Note!} \text{ true is as } \]

\( p \)-cappedness

Condition (1)

a solid \( \mathbb{Q}_p \)-module.

\( \mathbb{M} \otimes \mathbb{Q}_p \)
Harvey, can consider
dead.

decide complete (Solid)_{p}

then is closed under all limits, but not countable.

but (converse) closed under...-

\[ M \cap N_P = \min \{ M_{/p}, N_{/p} \} \]

\[ M_{/p} \cap N_{/p} \]

\[ \text{Eg.: } \]

\[ \begin{pmatrix} (A)_{2} \end{pmatrix} \uparrow_p \]

\[ = \begin{pmatrix} (A)_{2} \end{pmatrix} \uparrow_{p} \]

\[ \text{For ex. } S \rightarrow (2, \emptyset) \]
Then you take the tension product of this with
\[ T_{Nt} \text{ of } \text{U} (i) \text{ which is } \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \text{ depending on the context.} \]
Let's verify that the solid triangle exists. Need \( c_j \):

If we have a complex
\[
(M_0) = \ldots \rightarrow M_{-1} \rightarrow M_0
\]

where each term is \( \otimes \mathbb{H} \mathbb{C} \),

then we need

\[
\text{RHom}(\mathbb{H}, M_0) \cong \text{RHom}(\mathbb{H} \otimes \mathbb{C}, M_0)
\]

we did this already for \( M_0 \) in degree 0

\( \Rightarrow \) get it also if \( M_0 \) is a bounded complex.
Stupid funtions 

\[ M_{\geq n} \rightarrow M \rightarrow \text{something} \]

(Enough to show that \( R \) has \( \text{Ext}^1(\mathcal{O}_E, M) \) lies in degree at most e.g. \( N-2 \) \( \text{in} \) \( \mathbb{P}^N \).

Can shift \( M_{\geq n} \) to \( M_{>0} \).

\[ \Rightarrow \text{enough to see that} \]

\[ \text{Ext}^1(\mathcal{O}_E, M) = 0 \text{ for } i \geq 2. \]

Now take the Postnikov

functions

\[ M \rightarrow T_{\geq n}M \rightarrow T_{>0}M \rightarrow \]
Upshot: need

\[ \text{Ext}^i(\mathbb{IR}, \ker d_n) = 0 \]

\[ i \geq 2. \]

\[ \mathbb{T} \mathbb{Z} \]

\[ \text{Ext}^i(\mathbb{T} \mathbb{Z}, \ker d_n) = 0 \]

\[ i \geq 2. \]

\[ \mathbb{T} \mathbb{Z} \]

\[ \mathbb{T} \mathbb{Z} \]

\[ \text{Ext}^i(\mathbb{T} \mathbb{Z}, \ker d_n) = 0 \]

\[ i \geq 2. \]
\[ \text{The first exterior derivative is unique up to a } \mathbb{Z} \text{ isomorphism}. \]

\[ R_{\text{Hom}}(\mathbb{E}, \mathbb{F}) \xrightarrow{\sim} \mathfrak{g} \]

\[ \mathfrak{g} \text{ organize into a complex} \]

\[ M_0 \rightarrow M_0^R \rightarrow 1 \rightarrow M_0 \]

It's enough to show \( \text{Ext}^1(\mathbb{E}, \text{ker}(d_1^{R\mathfrak{g}})) \)

\[ \text{ker}(d_{R\mathfrak{g}}) \]

\[ \begin{array}{c}
\text{ker}(\mathbb{E}) \\
\mathbb{F} \end{array} \]