Solid $\mathcal{O}_p$-modules

\[ \text{Mod}_{\mathcal{O}_p}(\text{Solid}_p) \]
\[ \mathcal{O}_p \text{ proj. } \mathcal{O}_p \]
\[ \mathcal{O}_p \otimes \mathcal{O}_p = \mathcal{O}_p \]

\[ \Rightarrow \text{Mod}_{\mathcal{O}_p}(\text{Solid}_p) \cong \mathcal{O}_p \text{ proj. } \mathcal{O}_p \]

\[ M \otimes \mathcal{O}_p = M \otimes \mathcal{O}_p \]

\[ \mathcal{O}_p \otimes \mathbb{Z} = \mathcal{O}_p \]

\[ = (\mathcal{O}_p \otimes \mathbb{Z}) \left[ \frac{1}{p} \right] \]
\[ = \left( \mathbb{Z}_p \left[ \frac{1}{p} \right] \right) \]

\[ \Rightarrow \text{ proj. in Solid}_p \]

"Smith Spaces"
Banach spaces

\[ \bigoplus_{\mathbb{Q} \_p} \mathbb{Q} \_p \]

There are two duality between Smith pair & Banach spaces:

1) Duality:

\[ \text{Hom}_{\mathbb{Q}_p} \left( \prod \mathbb{Q}_p \mathbb{Z}_p, \mathbb{Q}_p \right) \]

\[ \cong \left( \bigoplus_{\mathbb{Q}_p} \mathbb{Q}_p \right) \]
21

Bench Span =

Not a major of Smith's Group

Space?

Might be possible

(potentially)

(probably)

Does not?

(?)

Hold's to

1.52

R2 = 0.25

R2H2O = 0.20
$$(\mathcal{O}(I))_{\mathcal{P}}^p$$

$$(\bigcap_{i \in I} \mathcal{O}(\mathcal{P})_{\mathcal{S}(\mathcal{A}_i)})^p$$

$$(\mathcal{P}_i)^p$$

$t : I \to \mathbb{R}$

tend to

$$0$$

$$\mathcal{P}_i$$-closed

of $\mathcal{O}(\mathcal{P})$

$$= \bigcap_{i \in I} \mathcal{O}(\mathcal{P})^p$$

**Note:** Sm"{a}tl spaces are not closed under colim.

**Claim:** quotients of Sm"{a}tl spaces $\mathcal{C}$

- compact objects
- in $\mathcal{C}$

- [Compact objects in a category]

- $\mathcal{C} [\frac{I}{P}]$

$\mathcal{C} :$ pro-$p$-abelian $\mathcal{P}$.
form on abelian subcategory.

$q \text{Smith} \cong q \text{Smith} \Rightarrow q \text{separated}.$

"non-separated Smith spaces".

\[
\left( \frac{f}{\mathbb{R}} \right)_{\mathbb{Z}/2}.
\]

Another for Banach as well:

\[ q \text{Banach} = \{ p \in \mathbb{S}_{\mathbb{R}} \mid 
\forall (x) \in M[\mathfrak{S}] \exists \epsilon > 0 \text{ s.t. } \forall \delta > 0 \text{ s.t. }
\]

\[
M_\delta \text{ solid, r.f.}
\]

\[ M \ni s \text{ complete, ring }
\]

\[ M(\mathbb{R}) \rightarrow M
\]

\[ \Pi_0(M(\mathbb{R})) \rightarrow M
\]
Rk: A separated solid $\phi_{\mathcal{D}}$

- unions of
  - Smill spaces

& all such are flat,

reunion of them

is closed under

i.e. $\phi_M$

\[ R_{\mathcal{D}} = \left( \bigcap_{\phi \in \mathcal{D}} \mathcal{C} \cap \frac{1}{\phi} \right) = \Pi \mathcal{C} \left[ \frac{1}{\phi} \right] \]
\[ \text{Tor}^i_{q_v}(M, N) = 0 \quad i > 1 \]

\[ M, N \in \text{C} \]

**Aim:** Frechet Spaces \( V, W \)

\[ C \quad V \preceq \lim_{n \to } \quad V_n \]

\[ \text{along dense inclusions maps} \]

\[ V \otimes W = \left( \lim_{n \to } V_n \right) \otimes \left( \lim_{m \to } W_m \right) \]

\[ = \left( \lim_{n \to } V_n \right) \otimes \left( \lim_{m \to } W_m \right) \]

\[ \text{tensor product} \]
Basic calculations:

\[
\left( \Theta \left( \frac{1}{2} \right) \right) \oplus \left( \Theta \left( \frac{1}{2} \right) \right)
\]

\[
\left( \Theta \left( \frac{1}{2} \right) \right) \oplus \left( \Theta \left( \frac{1}{2} \right) \right)
\]

Need an injection

\[
h : I \times I \rightarrow M_{2,2}
\]

tending to 0

is dominated by see

\[
\text{Coll} \cap \Gamma_0
\]
\( f(i) = \sqrt{\max_j h(i, j)} \)

\( g(j) = \sqrt{\max_i h(i, j)} \)

True for Baccch

\[
\prod_{N}^{Q_p} \bigotimes_{N}^{Q_p} \prod_{N \times N}^{Q_p} \bigotimes_{N \times N}^{Q_p} \bigcap_{N \times N}^{Q_p} \bigcup_{N \times N}^{Q_p} \bigcup_{N \times N}^{Q_p} \bigcup_{N \times N}^{Q_p}
\]

\( P : N - 1 \times R_2^0 \)

\( h : N \times (N - 1) \times R_2^0 \)

\( h \) dominate \( g \)

\( \exists e \in f \times g \)

Take

\( f(n) = \max \{ 1, \max_i h(i, n) \} \)
\( q(a) \cdot n \cdot \gamma \)

\[ V \circ T^n \]

Banach

\[ = TTV \]

This is the most elusive

\[ \exists \text{ fails on } \mathbb{R} \]

\[ 2 \bigotimes (2, 2) \]

\[ \neq T \bigotimes (2, 0) \]

(reduce mod \( p \))
I'll give an argument using concept of nuclearity.

Note: A small spore

\[ V = \left( \prod \mathbb{P} \right) \left( \frac{c_f}{\varepsilon} \right) \]

Can also be given a "Banach topology":

\[ V^B = \left( \prod \mathbb{P} \right) \left( \frac{c_f}{\varepsilon} \right) \]

In fact, \( V \subset V^B \)

\( \varepsilon \) natural on \( V \)

We have map

\[ V^B \xrightarrow{\varepsilon} V \]

\[ W \xrightarrow{f} V \]
Q: When does it factor this
√3 ?

A: f is deal
to a

spect speech
on Branch springs

f is tree-class.

Def: A tree-class map
W → V

is a map carrying some element of

\[(\text{Hom}(W, \mathbb{C}_p) \otimes V)_\mathbb{C}\]

via the natural map

\text{extension}
\[ \text{Hom} \left( W, \mathcal{O} \right) \otimes V \]

\[ \text{Hom} \left( W, V \right) \]

Reformulate:

Reformulation: \( V \in \text{Sol}_d \mathcal{O} \).

Define a contravariant \( \mathcal{O} \)-module \( \mathbf{V} \) by

\[ \mathbf{V} \left( s \right) = C \left( s, \mathcal{O} \otimes V \right) \left( s \right). \]

Then an\( \mathcal{O} \)-valued map

\[ \mathbf{V}^{\mathcal{O}} \to V, \]

& for \( W \) smooth

space, a \( \mathcal{O} \)-class map
\[ W \rightarrow V \]
\[ \Rightarrow \text{map factor for } V^\text{th} \]

\[ V \rightarrow V^\text{th} \]

\[ \text{Auto Solid } \rightarrow \text{Solid}_0 \]

Claim: \[ V \text{ Smith } \]

\[ \Rightarrow V^\text{th} = V_B \]

\[ \forall C \subseteq V \]

\[ \text{pf: } C(S, C_{p'}) \times (\prod_{p} (S_{p} \cup C_{p'})) \]

\[ = (C(S, C_{p'}) \times \prod_{p} (S_{p} \cup C_{p'})) \]

Let's use fact that
some p-complete sets are closed under $\otimes$.

\[
\left( \lim_{n \to \infty} \left[ C(S, \alpha^p_\omega) \otimes M^p_{\alpha^p_\omega} \right] \right)^{[f]} 
\]

\[
\left[ \lim_{n \to \infty} C(S, \alpha^p_\omega) \otimes M^p_{\alpha^p_\omega} \right]^{[f]} 
\]

\[
= C(S, \alpha^p_\omega) \left[ f \right]^{[f]} 
\]

\[
= C(S, \nu^3) 
\]

\[
\text{Nuclearity}
\]

Def.: $M$ is solid if

\[
\forall \varepsilon > 0 \exists t \in S
\]

\[
I \left( C(S, \alpha^p_\omega) \otimes M \right) (\varepsilon) \subseteq M(S)
\]
Equivalently, \( M^\text{th} \cong M \)

Hence, both sides are exact, i.e.,

\( \text{functors of } M \)

\( \text{of } \text{C}^\text{c}(X, Y) \)

\( \implies \text{Nuc} \) is

an abelian subcategory

closed under

co/limits.

Moreover, if we were
to make the

derived analog of

this definition,

derived nuclear

each still

is nuclear.

Thus, \( \text{M^n} \) is nuclear.
\( \implies M \text{ is a filtered colimit of } \mathcal{C} \text{ Banach spaces} \)

\[ \text{Nuclear = generated under \( \text{colim} \) by Banach spaces} \]

It's not an \( \mathcal{I} \Delta \)-category.

I.e. not exactly generated.

\[ \therefore \text{ it is } \mathcal{N} \text{-correctly generated} \]

Let the \( \mathcal{N} \)-correct object be also different from the Banach space!

\( \mathcal{N} \)-correct objects

- cokernel of
Basic nuclearity

\[ \lim_{n \to \infty} (V_0 \to V_1 \to V_2 \ldots) \]
trace class maps
spectral

Ric: Nuclear is independent if \( k > N_0 \)

(\text{every Banach space is } k\text{-condensed})

\[ \bigcup \text{banach spaces} \]

pf: First suppose \( M \)

\( \to \) Nuclear.

Write

\[ M = \lim_{i \to \infty} M_i. \]
Mr. compact objects
in a solid Qp.

\[ \text{flex } \sim \epsilon S \text{ Nk} \text{ span} \]

Note that

\[ M \sim M^+ \]

commutes w/ (cf. note)

So enough to note that

\[ = \] \[ M \sim \lim_{i \to} M_i^+ \]

\[ S_i \sim S_{oi} \to M_i \sim S \text{ NkH} \]

\[ = \]

\[ M_i^+ = \& \text{ Banach } \]

As

\[ (S \text{ NkH})^+ = \text{ Banach.} \]

For the course, it suffices to show that \( \& \) a Banach space
\((\mathcal{S}_r \otimes \mathcal{F}) \otimes V_r \mapsto v(s)\)

TENSOR PRODUCT OF BRANCH SPACES

\(\otimes\) v(S1)

CHOOSE BASIS & CALCULATE

Case: NUC Q C SOLID Q

It also closed under \(\otimes\).

Pf: Banach \(\otimes\) Banach = Banach.

This notion of nuclearity is in general cleaned in defined context.
(C, ∅) are combinatorial

∧ cptl + ser stable
ζ - czt

1 cptl object.

Some off these facts
hold in this context
others not.

e.g. Nuc N, cptl

\text{\textbf{Surprising fact:}}

Nuc clop cSolid

is closed under

countable products!
Re: This facts for $\Omega_p$

$\text{Nuc}_p = \text{filtered color}$

desired properties
of discrete part

$W_p$

but $\Omega_p$ is

$W$-

not nuclear

Ket case:

$\Omega_p \in \text{Nuc}_p$

if $\Omega_p = U (\Omega_p \cup \text{ext})$

Smith
$\tilde{\Pi}_{\mathcal{G}}(\mathcal{G}) \rightarrow \tilde{\Pi}_{\mathcal{G}}(\mathcal{G})$

$\mathfrak{R}^n \setminus \mathfrak{R}^n$ with coordinate $n$-th coordinate $= \text{basic example of a spect operator } \mathcal{G} \subset \mathcal{G}$.

General case:

$\prod_{n \in \mathbb{N}} M_n$

Write each $M_n = \text{filtered colim of } \mathcal{G}$ Brauch

$\Rightarrow \prod_{n \in \mathbb{N}} M_n = \left( \bigoplus_{n \in \mathbb{N}} \text{filtered colim of } \mathcal{G} \right)$

$\Rightarrow \prod_{n \in \mathbb{N}} M_n = \left( \bigoplus_{n \in \mathbb{N}} \text{filtered colim of } \mathcal{G} \right) \text{ Brauch}$
(Graphed ABG)

\[ T \text{ exact, so that reduces to} \]
\[ T \text{ Banach.} \]

But only for complete Banach spaces.

Need:

\[ TV \]
\[ V \]
\[ T \text{ Banach} \]

is nuclear.

Thus

Claim: \[ TV \cong \left( \prod \phi \right) \hat{N} \]

\[ \phi \in (S, \Phi) \]

\[ \text{PF: } V \cong (S, \Phi) \]
Let \( V = C \theta \).

By nuclearity of \( T \theta \), we have

\[
V \otimes T \theta = C(S, T \theta)
\]

\[
= T \theta C(S, T \theta).
\]

\[
(C(S, T \theta) \otimes M) \cong C(S, M) \otimes 1
\]

\[
= C(S, T \theta) \otimes M \cong C(S, M)
\]

because \( \rho \) is a projective closure over \( \Theta \).

Addendum: On Nuclearity

\[
V \otimes -
\]

comutes w/ countable products if

\( V \) is Banach.
If Same idea reducible

$V \otimes T W$

$\cong T V \otimes W$

For that use

$V \otimes T W$

$\cong (V \otimes W) \otimes T W$

$\cong T V \otimes W$. \qed

(om back to Frechet space):

Lemma (Mittag-Leffler):

$\exists (V_n)_{n \in N}$
a projective system of
Banach spaces with
dense transition
maps, then

\[ \lim_{n \to \infty} V_n = 0 \text{ (in Cond \(\mathcal{E}\))} \]

\[ 0 \rightarrow \lim_{n \to \infty} V_n \rightarrow \mathcal{A}V_n \rightarrow \mathcal{A}\mathcal{M} \rightarrow 0 \]

in Solid \(\mathcal{E}\).

reference to

\[ \mathcal{T} \mathcal{T} V_n \oplus \mathcal{T} \mathcal{T} W \]
\[ \mathbb{E} = \mathbb{T} \mathbb{W} \]
\[ \mathbb{T} \mathbb{W} = \mathbb{V} \mathbb{O} \mathbb{T} \mathbb{W} \]
\[ \mathbb{T} \mathbb{W} = \mathbb{W} \mathbb{T} \mathbb{W} \]

Decent

\[ A : \text{ comm alg on } \mathbb{Nuc}_0 \]

Define \( \text{Nuc}_A := \) \mod_A (Nuc(\mathbb{G}_p)).

\[ \text{Like could also look at } \mod_A(\text{solid } \mathbb{G}) \]

& redo the nuclear discussion
The result would be the same.

Then: $A \rightarrow B$

map of comm. algebras

in $\text{Nuc}_\mathbb{Q}_p$.

Suppose:

1) $\otimes B : \text{Nuc}_A \rightarrow \text{Nuc}_B$

has finite $\mathbb{Q}_p$-dimension.

2) $\text{Mod}_A(\mathbb{Q}_p) = \text{Mod}_A(\mathbb{Q}_p)$

as category.

$N_A = \text{Mod}_A(\mathbb{Q}_p)$

$M \in N_A$. 
If $M \in A$, consider
then so is $M$.

Then have descent:

$$N_A = \lim_{\to \infty} (N_B = N_{B^*})$$

Furthermore:

$$\text{Perf}(A \otimes B) \cong \lim_{\to \infty} (\text{Perf}(B^* \otimes B))$$

and (if $\text{Ext}^1(B^*, B) = 0$)

$$\text{Vec}(A \otimes B) \cong \lim_{\to \infty} (\text{Vec}(B^* \otimes B))$$

if $A \otimes B$ are "Fredholm".

$P \otimes A \rightarrow B_n$. 
map of $p$-torsors for $\gamma$ which is faithfully flat and $\eta$

$$= 1 \ A \mathcal{O}(\mathbb{P}) \ + \ B \mathcal{O}(\mathbb{P})$$

satisfies the hypothesis.

Conclusion = Thom of Drinfeld Vector

Matthew.

Sketch of proof:

Use Pearson-Reeves:

need that

$$\otimes_{\mathbb{A}} B: N_A \rightarrow N_B$$
1) Consuerce

\[ \lim_{n \to \infty} \left( \frac{\lim_{n \to \infty} X}{\Delta x_n} \right) \]

\[ \lim_{n \to \infty} X \]

\[ \lim_{n \to \infty} \]

\[ \lim_{n \to \infty} \]

\[ \lim_{n \to \infty} X \]

\[ \lim_{n \to \infty} \]

\[ \Omega \cdot B = \text{geom. rel. of } AB \triangle \]
General cat: [unlabeled cat]

Nuc n Specht

dualizable under $\otimes$

$X \xrightarrow{id} X$

has free cleft rep

$1 \xrightarrow{X \otimes X}$

is generated
\[
\text{cone}(M \xrightarrow{f} M)
\]
\[
\text{cpc+procede,}
\]
\[
f \text{ trace-class}
\]

Claim: \((\exists x) : G_p
\]
\[
\text{ & is Fredholm:}
\]
\[
\exists x \text{ such}
\]
\[
(M \xrightarrow{f} M) = x
\]

\[
i \text{ discrete}
\]

\[
\text{(relate to } G_p
\]
\[
X(\mathcal{A}) \subset G_p
\]
\[
\Rightarrow \text{ it's a perfect complex of } A
\]