Complex Analysis

Peter: $0 < p < 1 \quad \rightarrow \\
\text{lip}_p \in \text{Cond}_M$

abelian category

\& \quad \&

compact projective $\mathcal{M}_p(S), \, S \text{ equiv. } \mathcal{D}_C$

$C_l$: \text{Distain Arc}$

\text{liquid module}$

"complete enough"!

Test: Calculate some completed tensor products $\mathcal{M}$-complete c \text{ lip}_p \mathcal{A}_p
$V \otimes W$ is
not a Banach space
(not locally convex)
less complete than
$V \oplus W$

Same if $V \& W$
are small spaces

By definition, $S$ is a
$M_{c_p}(S) \otimes M_{c_p}(T)$
$= M_{c_p}(S \times T)$

Special case: $S = \bigcup_{n \in \mathbb{N}} \{a_n\}$

(continued...)
In functional analysis, we consider spaces $l^\infty$, $l^2$, and $l^1$. These structures are explored in the nuclear context.

Recall: there is a notion of "tree-dense" or "$\pi$-summing" maps of Banach spaces $V \rightarrow W$.

Example: $l^1 \rightarrow l^1$. 


\[ 2 \Delta a_1 \leq a \]

ant tree class E_t \text{, up}

\[ V \rightarrow W \]

\[ \begin{array}{l}
\text{Mg}

\text{rule:}

\text{one rootlet}

\text{Mg:} \quad \text{any p-summable}
\end{array} \]

\[ \text{map is p'-summable} \quad \text{for p' > p} \]

\[ \text{composite of } Z \]

\[ \text{p'-summable maps} \]
\[ 0 \leq p < \infty \]

\[ \bigvee \text{measurable} \]

\[ \text{affine} \text{ maps} \]

\[ A \text{ is a filtered union of Banach spaces over a class of injections.} \]

\[ A \text{ is a limit of Banach spaces along dense tree-class maps.} \]

\[ \text{Above discussion: you get the rare class of rare if} \]
Claim: $V W$ DNF space

$$V \otimes W = V \otimes W$$

(complete)

(see $V = U \otimes^{\sigma_p} \otimes_{\sigma_k}$)

Note: Same $V$ for NF spaces, but this is harder to see.

Ex: Rings of holomorphic
\[ D : \text{cond and think} \]
\[ C \]

\[ G(D) = f \text{ as holomorphic on an open neighborhood of } D. \]

**UNF Space**

\[ = \bigcup_{\varepsilon > 0} E(r, \varepsilon) \]

**Sequence Space, when \( E \) Shakes**

The transition maps are given by diagonal indices

\[ (1, t, t^2, t^3, \ldots) \]
\[ G(D_1) \otimes G(D_2) \]
\[ = O\left( \frac{1}{D_1 \times D_2} \right) \]

Let's consider the closed disks \( D_1 \) and \( D_2 \) contained in \( C \).

Then, contained in \( E \).

The \[ O(1) \otimes O(1) \]

\[ = O\left( \frac{1}{D_1 D_2} \right) \]
\( G(\overline{\delta}) \otimes O(\overline{\delta}) \)

\[ = \frac{G(\overline{\delta}) \otimes O(\overline{\delta})}{(\pi_{1,1})_{\mathcal{L}}} \]

\[ = O(\overline{\delta} \times \overline{\delta}) \]

\[ = O(\overline{\delta \times \delta}) \]

\[ = O(\overline{\delta \times \delta}) \]

Special case:

\( G(\overline{E}) \otimes O(\overline{E}) \)

\[ = O(\overline{E}) \]

\[ \Rightarrow \]

\[ \text{term on} \]

\[ G(\overline{E}) \]

\[ \text{same as on } C^T. \]
Special case: \( \alpha \wedge \beta = 0 \)

\[
\Rightarrow (\alpha(\mathfrak{d})) \& \alpha(\mathfrak{d}'(\mathfrak{d})) = 0.
\]

algebra reflects analytic at \( C \).

Where are we good?

Thm: \( X \) compact Riemann surface, \( E \) vector bundle \( \pi \).

Then:

1) \( H^i(X; E) \)

\( \& \& i \geq 2, \forall s. \)

\( \forall i, 0 \leq i \leq 2 \text{ (i21)} \)

2) \( H^0(X; E) \subseteq H^0(X; \mathcal{O}(E)) \)
For we'll prove it!

Prove a theory of quasi-coherent sheaves on \textit{C^*} manifolds with G-factors formalism.

\( = \) makes it easier to prove duality statements locally.

---

1) will come from:

\[ \text{Pic}(C^{\infty}) = \bigcap_{\text{cpt} \in C^{\infty}} \text{Pic} \]

2) can work locally.
A transform of shapes of complex space has to be defined.

Reason: \( \mathcal{O} \otimes \mathcal{G}(D) \) is not exact.

Proof: \( c_i = \text{const} \mathcal{G}(E) \) and \( \mathcal{G}(D) \) is not exact.

First up:

\( \mathcal{O}(E) \to \mathcal{O}(D) \)
We produced a 60 water function, ask for coherent cohomology in algebraic geometry. cf. induced pdf.

Generalized to idea of "analytic spine", new idea of "pure spine".

I'll take a half-classical approach.

Use usual underlay topological spine, put a sheet of confused my on it.

We'll build on the
Reminders on $\text{Sh}(X; \mathcal{O})$:

$$D(\text{Sh}(X; \mathcal{O})) = \text{Sh}(X; D(\mathcal{O}))$$

$$= \text{Sh}^\mathrm{h} \mathcal{O}(X; D(\mathcal{O}))$$

2 perspectives:
1) the classical theory of abelian categories
2) directly define the derived thing, derived analogy of classical thing.

\[ x \xrightarrow{f} y \]

\[ f^x : Sh(X, D(\mathbb{R})) \]

right adjoint

\[ f_* : Sh(C, D(\mathbb{R})) \]

\[ f_*(\mathbb{V} \circ Y) = f(\mathbb{V}) \]
$f^* f$ left adjoint:

pullback commutes

$(f^* f)_x = f(f(x))$

$f_y = \lim_{y \in V \searrow y} f(u)$

\underline{f^* c_{\text{cut}} = c_{\text{cut}}}


$X$ more subtle.

$X \xrightarrow{p} \text{pt}$

$p_\# \sim 2 \in \mathcal{O}(2)$

$\text{a complex}$
computing (sheaf!
cohomology of $X$.

$\mathbf{might}$ expect: $X - \rightarrow Y$,

$f^* \text{ is "in-fact" analog of cohomology.}$

But no! 

Thus $f: X \rightarrow Y$

proper $\Rightarrow$

\[
\begin{array}{cccc}
X & \xrightarrow{f} & X \\
\downarrow & & \downarrow \\
Y & \xrightarrow{g} & Y
\end{array}
\]

$g \circ f \simeq f \circ g$,

in particular: stalks of $L$ are $\text{colim of}$
\[
(f \circ g)(0) = \lim_\substack{u \to 0 \\ u > 0} f(g(u)) = \lim_\substack{u \to 0 \\ u > 0} g(u) = \lim_\substack{v \to 0 \\ v > 0} RT(v, 0; \theta) = \mathbb{Z} \cup \{0\} \oplus \mathbb{Z}
\]

So \( f \circ g \) is hyl.
Replacement: \( f_i \)

designed so \( f_{in} \) +

\( f_i = f_x \)

\( \geq f \text{ paper } \& \)

\( f_i \text{ always setters} \)

paper base change

(\& see w/ columns)

\( f_i = \text{ base } \& \text{ analog} \)

of sectors

w/ paper support

\( f : X \rightarrow Y \)

\( f \in \text{Sh}(X; D(A)) \)
\( f^* F(Y(V)) = \lim_{K \in \mathcal{K}} \mathcal{F}(X) - \mathcal{F}(X/K) \)

The right adjoint \( f^* \):

\( f^* \mathcal{F}(U) = \mathcal{F}\text{Hom}(f^* U, V) \)

Good for purity duality.

E.g.

Thm (Verdier): \( M \to \)

manifold. We have

\[ M \xrightarrow{f} \quad \text{we have} \]

\[ f^* V = f^* (f^* V) \circ f^* \]

If \( M \) is closed...
or \([d]\),
\[d = \dim M,\]

If: For an open set
\[U \subseteq M,\]
\[j^* = j^! = \cdot\]

\(\Gamma^\sim\) localizes on \(M\).

We check on \(\mathbb{R}^d\)
\[m + \rho^\sim = \rho^\sim,\]
then you are done.

\[\mathcal{T}_e(\mathbb{R}^d) = \mathbb{R}\]
\[\mathcal{T}_e(\mathbb{R}^d) = \mathbb{R}\]
\[ P \circ P' \circ P \]

\[ \pi \times P \circ \text{Prin}_2 \times P \]

\[ \pi \circ (M; \text{Out}) \]

\[ X: \text{Riemann surface} \]

\[ \text{Lurie: } \text{Sh}(X; \text{D}(\mathbb{Z})) \]

\[ \text{Sh}(\text{finite \coprod \text{ prof. sets}}; \text{D}(\mathbb{Z})) \]

\[ \text{topology of finite \coprod \text{ sets}} \]

\[ \text{CS: } \text{high \coprod \text{ Thom}} \]

\[ \text{class \ coproduct} \]
\[ f(K) = \lim_{K \to K'} \varphi(K') \]

\[ f \in Sh(X) \]

\[ f(K) = \lim_{U \supset K} f(U) \]

\[ = \mathcal{P}(i_{KX}^* f) \]

\[ \text{for } f \in Sh(\text{compact}) \]

\[ \implies f(U) = \lim_{K \in U} \varphi(K) \]

\[ \times \text{ Riemann surface.} \]

\[ U \subset X \sim \mathcal{O}(U) : \text{ring of holomorphic} \]
$S \rightarrow \mathcal{G}(U)$

$= \text{cts reps}$

$S \times U \rightarrow C$

set. $\uparrow$ holomorphic

$\exists \mathbf{u} \times U$

$(\mathcal{G})_{\mathbf{u}} \mathcal{C}^{\mathcal{G}_{\mathbf{u}}(U)}$

$G(D_{\mathbf{X}}) = \text{what is new here, our current hypothesis for } D_{\mathbf{X}}$

Consider the desired category
Consider sheaves of derived objects
$\mathcal{O}$-modules on $X$
with action of $\mathcal{O}(\cdot)$.

Define such an $\mathcal{O}$-module
sheaf $M$
if $A$

$D \subseteq D'$

the norms of closed disks,

$O(\overline{D}) \otimes M(\overline{D'})$

$O(\overline{D})$

$\Rightarrow$
We can define a function 
\[ f : X \rightarrow Y \]
only restricted in 2 cases:

\[ \left\{ \begin{array}{ll}
0 & \text{if true}
\end{array} \right. \]

occur exactly as in the
\[ \text{claim: they} \]
and $G$-modules $G$-modules and preserve quasi-commute.

$$(f \circ f)(s) = f^*f_{sa}.$$

So each $f$.

$f$ can be a column, be a height of point $e$. key $f$.

Proof:

Thus: $f'(v) = f^*(v) \circ \partial'd$

$f'(C) = \mathbb{R}(C)$. 

Key: Understand why (c)
Claim: $D \subseteq \overline{D}$

\[ Qcoh(D) \xrightarrow{\text{open disk}} \text{looks like.} \]

\[ Qcoh(D) \sim \frac{\text{Full subcategory}}{\text{st } G(D)} \]

\[ \text{of } G(D) \text{-modules in } \text{stabilized } C\text{-modules.} \]

\[ M \text{ st. } M \otimes G(D) \]

\[ G(D) \]

\[ E \in \mathcal{D}, \quad E + M \]

\[ \mathcal{F}(E) = M \otimes G(E) \]
\[ f \left( j; \mathcal{A} \right) \]

then

\[ \mathcal{A} \left( \mathcal{A}(D) \right) \to \mathcal{A}(D) \otimes \mathcal{A}(D) \]

Rec: \[ G(D \times D) \]

is an idempotent

\[ G(D) - \text{algebra} \]

\[ \mathcal{Q}(\mathcal{G}(D)) \]

gotten for \( \mathcal{G}(D) \)

let killing ac
important section

1) description of

$P \times \{ \overline{P} \} \cap \overline{P}$

$\phi \cdot V = \frac{\partial H_{\mu} \left( S_{\mu} \left( \frac{\nabla \left( \Phi(0) \right)}{\Phi(0)} \right) \right)}{\partial \mu}$

$\phi \cdot V = \frac{H_{\mu}}{g(5)} \left( \frac{\partial}{\partial \mu} \left( \Phi(0) \right) \right)$

reduced to calculate

$\Phi(0) \left( \Phi(0) \right) \cdot g(5)$

= 1

1 comes on "complementary"

"closed disk in $P"$

"$P$"
$p^i = p^* \otimes \mathcal{L}^i$ 

Cor. More that $\prod$ counts $wl$ columns, i.e. right adjoint of $p^i$ counts $wl$ all columns.

$= \otimes \text{really } p^i$

snd1 cpc objects to cpc cpc objects.

vector bundle is dualizable, hence is compact $\iff O(1)$.
if $X$ compact, get that

$$\mathcal{F}(X; \mathcal{U}) = \mathcal{U} \mathcal{V} = \mathcal{V} \mathcal{V}$$

ii) a compact object in Qgrp.

Ooh, this $\mathcal{F}$ is a dense free limit of $G(\mathcal{B})$s

$$\Rightarrow \text{ "nuclear" rigid module}$$

Thus we use
\[ \text{Nuclear} + \text{PC} = \text{Project} \]
\[ \Rightarrow \text{finite dual,}\]
\[ \text{finiteness of} \]
\[ \mathcal{B}(X;N) \]
\[ = \text{Sen doulitt} \]
\[ = \text{just as in} \]
\[ \text{Verdon setting} \]

Recall: \( \mathcal{M} \) \text{Nuclear} \[ \Rightarrow \forall \mathcal{S} \text{exh} \]

\[ \mathcal{C}(S, C) \otimes \mathcal{M}/(S) \rightarrow \mathcal{M}(S) \]

\[ \Rightarrow \text{Banch space are} \]
\[ \text{wrt flat, wrt} \]
We don't know water to closed under $\mathbb{W}_1$.

(1) See in eq. colors by "basic nuclei"$
\lim \left( \frac{m_0 - m_c}{n - m} \right) \uparrow \uparrow\uparrow$

get the class

Special case: $\rightarrow$ as

=) DNF.

Recall:

<table>
<thead>
<tr>
<th>nuclei</th>
<th>nucleon</th>
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<tbody>
<tr>
<td>$\text{Dalitz} \subseteq$</td>
<td>compact</td>
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$\forall Y + \cos \left( m \left( \frac{-f}{r}, m \right) \right)$