Graduate Student Meeting on Applied Algebra and Combinatorics 2021

List of Abstracts

Graded algebras with cyclotomic Hilbert series

Alessio Borzì

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The numerator of the Hilbert series of a complete intersection graded k-algebra is easily expressed as a product of binomials of the form $(1 - t^n)$. Therefore, the numerator (also known, in the standard graded case, as h-polynomial) has all its roots in the unit circle, that is, it is the product of cyclotomic polynomials. In one of his papers, Stanley asks if this condition suffices to characterize the Hilbert series of complete intersections. He immediately provides an easy counter-example (precisely $k[x, y]/(x^3, xy, y^2)$). In this joint work with Alessio D'Alì, we investigate this question in some families of algebras, such as numerical semigroup rings, Koszul algebras and polytopal algebras, providing a general framework for many apparently different results in relation to a conjecture involving cyclotomic numerical semigroups and techniques for Ehrhart positivity, as well as providing new results for Koszul algebras.

Algebraic and semi-algebraic phylogenetic reconstruction

Marina Garrote-López

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To model evolution, one usually assumes that DNA sequences evolve according to a Markov process on a phylogenetic tree ruled by a model of nucleotide substitutions. This allows to define a distribution at the leaves of the trees and one might be able to obtain polynomial relationships among the probabilities of different characters. The study of these polynomials and the geometry of the algebraic varieties defined by them can be used to reconstruct phylogenetic trees. However, only a subset of these algebraic varieties has biological sense. In this talk, we will discuss the importance of studying these subsets and we will prove that, in some cases, considering them is fundamental for the phylogenetic reconstruction problem. Finally, we will present the phylogenetic quartet reconstruction method ASAQ which is based on the algebraic and semi-algebraic description of distributions that arise from the general Markov model on a quartet tree. Its performance on multiple types of data will be discussed and it will be compared with classical phylogenetic reconstruction methods.

Exact Solutions in Log-Concave Maximum Likelihood Estimation

Alexandros Grosdos

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In nonparametric statistics one abandons the requirement that a probability density function belongs to a specific statistical model. Given a sample of points, we want to find the best log-concave distribution that fits this data. This problem was originally studied by statisticians, who found that the optimal solutions have probability density functions whose logarithm is piecewise-linear and used numerical methods to approximate the pdf. Our goal is twofold. On one side we use algebraic and combinatorial methods to explore in which cases we can find exact solutions to this problem. At the same time we use tools from algebraic geometry to test if the solutions provided by statistical software can be certified to be correct.

This is joint work with Alexander Heaton, Kaie Kubjas, Olga Kuznetsova, Georgy Scholten, and Miruna-Stefana Sorea.

Cohen-Macaulay binomial edge ideals and accessible graphs

Antonio Macchia

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Binomial edge ideals are binomial ideals generated by the 2-minors of a $(2 \times n)$-generic matrix corresponding to the edges of a graph. Their minimal primary decomposition is encoded by special sets of vertices of the graph whose removal disconnect the graph, called cut sets. An interesting open problem consists in finding a characterization of Cohen-Macaulay binomial edge ideals. To this aim, we introduce the class of accessible graphs as the graphs with unmixed binomial edge ideal and whose cut sets form an accessible set system. We prove that the graphs whose binomial edge ideal is Cohen-Macaulay are accessible and we conjecture that the converse holds. We settle the conjecture for large classes of graphs, including chordal and traceable graphs, providing a purely combinatorial description of Cohen-Macaulayness.

Rational general solutions of first-order algebraic ordinary differential equations

Johann Mitteramskogler

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Algebro-geometric methods have been utilised recently to find certain types of solutions of algebraic differential equations (ADEs). By neglecting the differential aspect at first, an ADE may be considered as the defining relation of an algebraic variety. If this variety is parametrisable by functions from the class of solutions one is interested in, then this parametrisation may be transformed into one which respects the differential aspect as well. The latter then constitutes a solution of the original ADE. This talk will introduce two methods for constructing rational general solutions of first-order ordinary ADEs following the aforementioned approach. A key step in both methods is the computation of a proper rational parametrisation of the obtained variety which leads to an associated (system of) differential equation(s). This construction is such that solutions of the associated system/equation can be transformed immediately to solutions of the original ADE and vice versa. Furthermore, explicit solution methods for solving the associated equation(s) are known. In closing, some connections between certain classes of parametrisable first-order ordinary ADEs and those possessing a rational general solution are investigated more closely.

Singularities in visual servoing of five points using symmetries

Abhilash Nayak

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Image-based Visual Servoing (IBVS) is the control of a robot by observation of geometric entities such as 3D points on a fixed target object and using their projected image on the camera attached to the robot. It includes a mapping between two vector spaces: the time variation of the projected 2D points on the image plane of the camera and the instantaneous spatial velocity of the camera attached to the robot. The configuration of the camera and the points is said to be singular if the matrix that maps the two vector spaces, the so-called interaction matrix loses rank. Based on the results for IBVS of three and four points, the singularities in the IBVS of five points amounts to finding the intersection between 10 cylinders, where each cylinder passes through three of the five points with its axis normal to the plane containing those three points. The goal is to determine the conditions on the affine coordinates of the five generic points such that the intersection of those 10 cylinders is a non-empty set. To do so, the Abelian group actions on the coordinates of the points such that the ideal of the cylinder equations remains invariant are determined. Thus, a linear transformation is applied to those coordinates to further split the equations still generating the same ideal. Finally, Abelian F5 Groebner basis algorithm will be implemented to determine the confiuguration of five points for the occurrence of singularities in the IBVS.

A central limit theorem for the two-sided descent statistic on Coxeter groups

Frank Röttger

Université de Genève

Coxeter statistics are maps which assign natural numbers to each element of a Coxeter group. When we pick an element at random, for example according to the uniform distribution, a Coxeter statistic generates a random variable. A sequence of uniformly chosen elements from Coxeter groups of growing rank therefore gives rise to a sequence of random variables. We study the statistic which assigns to an element of a finite Coxeter group the number of descents plus the number of descents of its inverse. Our main result shows that this statistic is asymptotically Gaussian when the rank of the non-dihedral irreducible components tends to infinity. This is achieved by a combination of already known results for the irreducible types (e.g. the symmetric and hyperoctahedral groups) and an application of Lindeberg's theorem. Our work answers a question of Kahle–Stump and generalizes work of Chatterjee–Diaconis, Özdemir and Röttger. This is joint work with Benjamin Brück.

A combinatorial approach to Minkowski tensors of polytopes

Amy Wiebe

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Intrinsic volumes of a convex body provide scalar data (volume, surface area, Euler characteristic, etc.) about the geometry of a convex body independent of the ambient space. Minkowski tensors are the tensor-valued generalization of intrinsic volumes. They provide more complex geometric information about a convex body, such as its shape, orientation, and more.

Minkowski volume tensors are closely linked to the moments of the uniform distribution on a convex body, and a rational generating function for these moments allows us to extract the tensors symbolically. In this talk, we explain this connection and show that it can be extended to the setting of Minkowski "surface tensors". We demonstrate how this generating function approach allows us to give an explicit formula for these surface tensors in the case of simplicial polytopes.

No prior knowledge of Minkowski tensors will be assumed.

This is a joint work with Büşra Sert and Niklas Livchitz.