# BIG PURE PROJECTIVE MODULES OVER LOCAL COMMUTATIVE NOETHERIAN RINGS: COMPARING WITH THE COMPLETION

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There is a big amount of work done on the study of finitely generated (hence, finitely presented) modules over commutative noetherian rings, but not so much is known on direct summands of infinite direct sums of such modules. With our work we want to give some insight into this problem.

Let $R$ be a local commutative noetherian ring. If $R$ is complete then all pure projective modules are direct sums of indecomposable finitely generated modules with local endomorphism ring. Therefore such decompositions are quite well behaved and satisfy the the infinite version of the Azumaya-Krull-Schmidt Theorem. This is not true in the non-complete case, even in the case of rings with finite Cohen-Macaulay type, there are countably generated pure-projective modules that are neither direct sums of finitely generated modules, nor direct sum of indecomposable modules.

There is still a close relation between pure-projective $R$-modules and pure projective $\hat{R}$-modules, as we can prove that two pure-projective modules $P$ and $Q$ are isomorphic as $R$-modules if and only if $P \otimes_R \hat{R} \cong Q \otimes_R \hat{R}$ as $\hat{R}$-modules. The proof of such result is modelled on Příhoda's proof that two arbitrary projective modules are isomorphic if and only if they are isomorphic modulo the Jacobson radical [3].

The difficult question is to determine which pure-projective $\hat{R}$-modules are extended from $R$-modules. When $R$ is a one dimensional domain we are able to give an infinite version of a very userful criteria due to Levy and Odenthal. Such criteria allows us to examine the construction in [6] (see also [2]) which give plenty of pathological examples.

Our techniques use heavily the results in [1] and some papers by P. Příhoda and G. Puninski [4] [5] that were showing the way towards the pure-projective case.

The results that will be presented are part of an ongoing long joint project with Pavel Příhoda and Roger Wiegand.

## References

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