Stochastic Programming and its applications

Giovanni Pantuso

I work on Operations Research and, particularly, on *optimization*. An optimization problem is

$$\min_{x \in \mathcal{X}} \Big\{ f(x) : g_i(x) = 0, i = 1, \ldots, I \Big\} \tag{1}$$

where $\mathcal{X}$ is a subset of $\mathbb{R}^n$, $f(x)$ and $g_i(x)$, $i = 1, \ldots, I$ are scalar functions of x. That is, we want to make decisions x that minimize an objective function $f(x)$ while respecting a number of restrictions expressed by $g_i(x) = 0$. When the functions $f(x)$ and $g_i(x)$ are linear in x we obtain a *linear programming problem*. When $\mathcal{X}$ forces some of the x variables to take integer values, e.g., binary, we obtain a *mixed-integer (linear) programming problem*.

In practical cases, decisions x are often made before the data that specifies $f(x)$ and $g_i(x)$, $i = 1, \ldots, I$ becomes known (e.g., we produce before knowing demands, we invest in assets before knowing their returns). In these cases, the parameters that define these functions are better represented by random variables. We obtain a so called *stochastic programming problem*

$$\min_{x \in \mathcal{X}} \Big\{ f(x, \boldsymbol{\xi}) : g_i(x, \boldsymbol{\xi}) = 0, i = 1, \ldots, I \Big\} \tag{2}$$

where $\boldsymbol{\xi}$ is a random variable with known probability distribution (typically independent of x). A common specification of (2) is that where $f(x)$ takes the form of an expectation

$$f(x) := \mathbb{E}_{\boldsymbol{\xi}}\big[F(x, \xi)\big]$$

where $F(x, \xi)$ is a scalar function of x and ξ (a realization of $\boldsymbol{\xi}$). The resulting formulation makes sense in many practical cases such as when the decision process repeats itself over time. Then, by the Law of Large Numbers, for a given x, the average of the total result over many repetitions will converge to the expectation $\mathbb{E}_{\boldsymbol{\xi}}\big[F(x, \xi)\big]$, and the solution of problem (2) will be optimal on average. Alternative settings include minimizing risk measures, or enforcing that certain events happen with a desired probability.

I am currently looking at two general issues. The first, is that of obtaining a more tractable formulation and solution algorithm for sequential decision problems with *endogenous* uncertainty, that is, where decisions x influence the probability distribution of $\boldsymbol{\xi}$ [1]. These problems, which arise for example in the petrochemical industry [3], are currently much more involved than the form (2). They give rise to extremely large *scenario trees* (a structure that describes the evolution of the uncertainty over time) and produce formulations with far more decision variables and constraints that modern calculators and solvers can handle. I am currently studying new scenario tree structures that reduce the size of the corresponding formulations.

I am also studying improvements to existing solution algorithms for *two-stage* mixed-integer linear stochastic programs, that is decision problems where certain decisions have to be made prior to the resolution of the uncertainty $\boldsymbol{\xi}$ and a set of corrective actions can be taken after the uncertainty has resolved. In a nutshell, while existing algorithms (i.e., Benders decomposition) separate current and future decisions, the general idea I am exploring is that of anticipating some of the future decisions in order to obtain faster convergence to high quality current solutions. A proof of concepts is available in [2].

Finally, I am working on a number of practical applications of (stochastic) optimization. Examples of these are problems emerging: (i) in the operational management of modern carsharing systems [4], (ii) in football transfer market decisions [6] and (iii) in the installation and maintenance of offshore wind farms [5]. This is joint work with Kjetil Fagerholt, Henrik Andersson and Magnus Staalhane (NTNU, Trondheim) and Lars Magnus Hvattum (Molde University College).

References

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