My research

What do “Erfindungskunst”, World War II, and the “power of a superman” have to do with mathematics? The first is from a quote by David Hilbert (1862-1943) from his memorial speech for his friend and colleague Hermann Minkowski (1864-1909), referring to Minkowski’s proof through “einer geometrisch anschaulichen Betrachtung” for an upper limit for the minimum value, M, of a positive definite quadratic form for integer values of the variables not all zero. The last one was wishful thinking by Nicolas Rashevsky who set up the first research group in biomathematics in Chicago. The Second World War changed funding and images of science. Together, these three expressions in some sense span my research interests in the history of modern mathematics, pure and applied, their inter-connections and the broader socially determined conditions for mathematical research.

I study the history of mathematics from perspectives of past mathematicians, their projects and motivations, situated in certain contexts, at specific places, at certain times, and under particular historical circumstances in order to understand and explain historical processes in the development of mathematics. My approach is to study concrete episodes from the history of mathematics within the “work place”, so to speak, of the involved (past) mathematicians, studying the development of these mathematicians’ production of mathematical knowledge, or uses of mathematics, from their practice(s), trying to follow the development of their ideas and techniques. It involves asking and answering questions such as why mathematicians introduced specific definitions and concepts, why they worked on the problems they did and how, which techniques did they use and why. The aim is to get insights into issues such as how mathematical objects emerge and develop, what kind of factors, and driving forces (internal as well as external to mathematics), and actions have modified the course of mathematics.

One such episode is the development of the concept of a general convex body which was coined by Minkowski at the turn of the century. Eighty years later Werner Fenchel (1905-1988) wrote in his paper “Convexity through the ages” that: “Minkowski’s interest in convexity originated, strange to say, from the Theory of Numbers” signaling that by then, convexity was disconnected from number theory. Minkowski came to convexity through the way in which he approached the minimum problem for positive definite quadratic forms in n variables. The minimum problem is to find the minimum value of such a quadratic form

$$f(x_1, \dots, x_n) = \sum_{h,k=1}^{n} a_{h,k} x_h x_k, \qquad a_{h,k} \in \mathbb{R}, \qquad a_{h,k} = a_{k,h}$$

for integer values of the variables $(x_1, \dots, x_n)$ not all zero. This minimum is essential in the reduction theory of positive definite quadratic forms.

The problem was not new at Minkowski’s time. Joseph-Louis Lagrange (1736-1813) had addressed the problem for forms in two variables in the eighteenth century. Johann Carl Friedrich Gauss (1777-1855) had realized its connection to the question of so-called reduced forms at the turn of the century, and Charles Hermite (1822-1901) had studied the minimum problem and its connection to reduced forms of an arbitrary number of variables. The new contribution of Minkowski's work on the minimum problem was not the statement of the problem itself but his approach. He relied on geometrical intuition for dealing with the minimum problem in *n* variables and used a geometrical framework to discuss the minimum problem. The idea of interpreting positive definite quadratic forms geometrically was not Minkowski's own. Gauss had presented an outline of such a geometrical interpretation in 1831 for the case of two variables, and Dirichlet (1805-1859) had shown how to represent positive definite quadratic forms in three variables geometrically. In a rectangular coordinate system, the level curves of a positive definite quadratic form in two variables form ellipses. Through a certain coordinate transformation, the level curves form circles, and the square of the Euclidean distance from the origin to a lattice point (a point with integer coordinates in the (skew) coordinate system) equals the value of the quadratic form for these integer values of the variables. Hence, the minimum problem becomes the problem of finding the smallest distance in the lattice. Minkowski generalized the geometrical approach to positive definite quadratic form in *n* variables, and he was able to find an estimate for the upper bound of the quadratic form for integer values of the variables by a simple comparison of two volumes.

At some point, probably around 1891, Minkowski realized that the essential property for his argument for comparing the two volumes was the property of convexity. He introduced a lattice consisting of the points in Euclidean *n* space with integer values for the coordinates. He defined what he called a radial distance function *S* and its corresponding unit ball, or "Eichörper", as he named it, consisting of all points *u* for which the radial distance to the origin is less than or equal to 1. Minkowski's ideas of radial distance functions and "Eichkörper" were presented at a talk that was read at the international Mathematical Congress at Chicago in 1893. In the talk, he restricted himself to three dimensions but announced that the theory would be fully developed for *n*-dimensions in his forthcoming book *Geometrie der Zahlen*. He argued that the "Eichkörper" of a radial distance for which the triangular inequality holds is convex, or nowhere concave as he called it at that time, and vice versa that every nowhere concave body, which has the origin as an inner point, is the "Eichörper" of a certain radial distance for which the triangular inequality holds. A mathematician, reading Minkowski's talk today will recognize a reciprocal radial distance function *S*, for which the triangular inequality holds, as a metric that satisfies the conditions for $(\mathbb{R}^3, S)$ to also be a normed space. Minkowski reformulated his result for the minimum problem into his famous lattice point theorem, here quoted from *Geometrie der Zahlen*:

> Ein nirgends concaver Körper mit einem Mittelpunkt in einem Punkte des Zahlengitters und von einem Volumen = 2*n* enthält immer noch mindestens zwei weitere Punkte des Zahlensgitters, sei es im Inneren, sei es auf der Begrenzung.

Minkowski then began to study convex bodies for their own sake, publishing several papers on convexity where he worked on convex bodies completely detached from quadratic forms and number theory as such. He gave a systematic treatment of convex bodies in 3 dimensions, introduced the now standard notions of distance function for convex bodies with the origin as an inner point, supporting hyperplanes, separating hyper planes, mixed volumes etc.

From there on the number of manuscripts and materials on convex bodies or using convex bodies multiplied, and around 1932 Otto Neugebauer (1899-1990) suggested that Tom Bonnesen (1873-1935) and Fenchel should collect, organize and systematize all of this. The result of their joint effort was the book *Theorie der konvexen Körper*. The appearance of Bonnesen's and Fenchel's monograph can be seen as a first step of the formation of convexity as a (sub)discipline of mathematics.