My Research
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My main mathematical interests lie at the interface between operator algebras (von Neumann algebras, C*-algebras and operator spaces), noncommutative probability (in particular, L_p-theory of noncommutative martingales), and group theory (more precisely, some of its analytic, geometric and probabilistic aspects).

The theory of operator algebras was founded in the 1930's by Murray and von Neumann to establish a mathematical frame for quantum mechanics, in which, for example, Heisenberg's uncertainty relation is expressed as non-commutativity of certain operators. Von Neumann algebras are the natural framework for noncommutative measure theory and integration, while C*-algebras can be thought-of as non-commutative topology. Operator spaces are a natural quantization of Banach spaces. An operator space E is a Banach space given together with an isometric embedding into $\mathcal{B}(H)$, the algebra of bounded linear operators on a Hilbert space H. For all positive integers n, this embedding induces a norm on $M_n(E)$, the algebra of $n \times n$ matrices over E, determined by the isometric identification $M_n(\mathcal{B}(H)) \cong \mathcal{B}(H^n)$. In this way, a Banach space is given a structure resembling the one of a C*-algebra, but in contrast, it may admit many non-equivalent operator space structures. The morphisms between operator spaces are completely bounded maps. Given a linear map u between operator spaces E and F, we say that u is *completely bounded* if the corresponding amplification maps $u_n : M_n(E) \to M_n(F)$ are bounded uniformly in n.

In 1995, I became a graduate student at the University of Illinois at Urbana-Champaign. I came from Romania, with a background in classical analysis and probability, and the desire to pursue research in Banach spaces. A year later, I enrolled in a course that Z.-J. Ruan (one of the founders of operator space theory) taught for the first time in Urbana. He was organizing the material for the writing of his monograph together with E. Effros, that later became a fundamental textbook on the subject. I was fascinated with the beautiful interplay between classical and noncommutative straight from the beginning. That same year, I was also introduced to the magical world of D. Burkholder's mathematics, and his powerful use of martingale theory (building on fundamental work of J. Doob) in shedding light on the deep interconnections between probability and various fields of analysis. His work on the UMD (*unconditionality for martingale differences*) property, introduced by Maurey and Pisier, had a tremendous influence on harmonic analysis in the Banach space-valued setting.

In 1997 Pisier and Xu proved the noncommutative analogue of the classical Burkholder-Gundy square function inequalities. Here the probability measure is replaced by a normalized trace τ on a von Neumann algebra $\mathcal{M}$, the filtration becomes an increasing sequence $(\mathcal{M}_n)$ of von Neumann subalgebras of $\mathcal{M}$, the space $L_p(\mathcal{M}, \tau)$ is defined as the completion of $\mathcal{M}$ equipped with the norm $\|x\| = \tau(|x|^p)^{1/p}$, and the conditional expectation with respect to $\mathcal{M}_n$ is the orthogonal projection from $L_2(\mathcal{M}, \tau)$ onto the closure of $\mathcal{M}_n$ in $L_2(\mathcal{M}, \tau)$. They also showed the noncommutative analogue of the celebrated Fefferman-Stein duality between the Hardy space H_1 and BMO of martingales. Burkhoder found the results extremely interesting, and decided to explore martingale inequalities in the noncommutative setting. Under his guidance (joint with M. Junge), I then started my thesis research on vector-valued noncommutative L_p-spaces. Stopping time arguments and maximal functions, which are crucial tools in probability, appear unavailable in the noncommutative setting, since the pointwise supremum of a sequence of bounded linear operators may not necessarily represent a (possibly unbounded) linear operator at all. Therefore, functional-analytic methods such as complex interpolation, as well as appropriate concepts provided by the theory of operator spaces, become very important tools. My first thesis result established the fact that, just like in the classical setting, the noncommutative BMO is a natural substitute for L_∞ as an end-point for interpolation with an L_p-space. The question whether an analogue of property UMD$_p$ for operator spaces, defined by Pisier, holds independent on p, has remained open. It is a problem that I return to from time to time.

A postdoctoral position at SDU, Odense, in 2006, marked the beginning of a long and fruitful collaboration with U. Haagerup, that resulted in five published papers, and an unfinished manuscript at the time of his tragic passing away last Summer. Together we obtained new proofs with improved constants of the Khintchine-type inequalities with matrix coefficients (due to Pisier-Lust-Piquard, respectively, Junge), leading to a new proof of a completely isomorphic embedding of Pisier's operator Hilbert space OH into a noncommutative L_1-space, we established a classification of hyperfinite factors up to completely bounded isomorphism class of their preduals, and solved a conjecture, due to Effros and Ruan, asserting that a certain Grothendieck-type inequality for a bilinear form on C*-algebras holds if (and only if) the bilinear form is jointly completely bounded. Furthermore, by investigating the class of completely positive maps between von Neumann algebras posessing a certain factorizability property, we solved (in the negative) an open problem in quantum information theory. Namely, we showed that a conjectured restoration in the asymptotic limit of the classical Birkhoff theorem asserting that any unital quantum channel can be well approximated (in cb-norm) by unitarily implemented ones, fails in every dimension. We moreover established a reformulation of the Connes embedding problem in terms of a certain asymptotic behaviour of factorizable quantum channels.

More recently, I have become very interested in exploring the close relationship between groups and their associated operator algebras. In very recent work with R. Grigorchuk and M. Rordam, we investigate questions concerning locally finite groups, just infinite groups and C^*-algebras, and connections to properties of Grigorchuk's group of intermediate growth. I am also interested in studying Poisson-Furstenberg boundaries associated to random walks on groups, and also plan to pursue the recently emerged applications of embeddability of expanders to rigidity of group actions on Banach spaces and non-exactness of groups.