My research: Energy optimization under uncertainty and competition

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My research interests revolve around applications of mathematical programming to the energy sector. This involves constrained optimization and other decision-making models for investment and operations management and how to adapt these models to the organizational and technological development.

Unit commitment

A classical problem in discrete optimization is the so-called Unit Commitment problem. We consider a set of power producing units and a discrete planning horizon, indexed by $\{1,\ldots,I\}$ and $\{1,\ldots,T\}$, respectively. The problem is to determine the online status u_{it} and corresponding production level q_{it} of each unit and at every point in time such as to maximise social welfare, meet total demand for electricity in the system and comply with the technical constraints of the units. Formulated by mixed-integer convex quadratic programming, the problem is

$$\max \Big\{ \sum_{t=1}^{T} \Big(\int_0^{d_t} p_t(s)ds - \sum_{i=1}^{I} (a_i u_{it} + b_i v_{it} + c_i(q_{it})) \Big) : q_i^{min} u_{it} \le q_{it} \le q_i^{max} u_{it}, \ \ , u_{it} - u_{it-1} \le v_{it},$$
$$\sum_{i=1}^{I} q_{it} = d_t, \ \ u_{it} \in \{0,1\}, v_{it} \ge 0, \ \ i = 1,\ldots,I, t = 1,\ldots,T \Big\},$$

The objective function accumulates consumer and producer surpluses, where $p_t(\cdot)$ is a linear inverse demand function, $c_i(\cdot)$ is a convex function capturing variable production costs, the parameter a_i is a fixed production cost and b_i is a start-up cost. The constraints ensure that the production level does not exceed its lower and upper bounds q_i^{min} and q_i^{max} when a unit is online; that start-up costs are included when a unit changes status from offline to online, and finally; that demand and supply balances at every point in time. The mathematical programming framework makes it easy to formulate other technical and/or logical constraints such as ramping restrictions (bounds on the rate of change of production), minimum up-times and down-times (if a unit is online/offline, it has to remain online/offline for a certain period of time), down-time dependent start-up costs and network constraints. Moderately sized problems can be solved by standard Branch and Bound (a numerical enumeration approach that exploits bounds to exclude subsets of the solution space), whereas the solution of real-life instances often requires decomposition with respect to units and/or time periods.

Competition

Since the 1990s, widespread efforts have been made to deregulate the electricity markets. As a result, investment and operating decisions are no longer made by a system operator but by profit-maximizing power producers trading through an electricity exchange. Under perfect competition, deterministic market prices and convexity of the objective function and the constraints, the market equilibrium can be shown to maximize social welfare. This does not hold when producers are able to exercise market power. If, for instance, producers compete á la Cournot (i.e. in quantities only) and ignore fixed costs, the market equilibrium is obtained as a solution to the system

$$\max \Big\{ \sum_{t=1}^{T} (p_t(q_{-it}, q_{it}) q_{it} - c_i(q_{it})) : q_i^{min} \le q_{it} \le q_i^{max}, \ \ t = 1,\ldots,T \Big\} \ \ i = 1,\ldots,I,$$

$$\sum_{i=1}^{I} q_{it} = d_t, \;\; p_t = \beta_t - \alpha_t d_t, \;\; t = 1, \ldots, T,$$

where q_{-it} is the vector of production levels $q_{i't}$ for producers $i' \neq i$ and $\partial q_{-it}/\partial q_{it} = 0$. By deriving optimality conditions for the profit-maximization problems, the equilibrium problem becomes a mixed-complementarity problem that can be reformulated and solved by mixed-integer convex quadratic programming.

Uncertainty

To reduce the dependency of the power system on fossil fuels, many industrialized countries seek to increase the deployment of renewable energy sources. In contrast to conventional generation that can start up, shut down and ramp, renewable generation is often intermittent, meaning it cannot be controlled but varies stochastically over time. From a producer perspective, the higher variation in supply is reflected in the market price. Profit maximization therefore requires an appropriate representation of uncertainty. By assuming a discrete distribution of the random parameters, stochastic programming allows for decision-making under uncertainty as a direct extension of deterministic mathematical programming. In recent joint work with Salvador Pineda Morente (at KU until recently) and Sonja Wogrin, we consider the joint problem of investment and operation. Investments are made with only distributional information on market prices, whereas operational decisions can adapt to their realizations. Given a number of realized price paths indexed by $\{1, \ldots, S\}$ and their probabilities π^s, the producer's problem is

$$\max \; \Big\{ \sum_{s=1}^{S} \pi^s \sum_{t=1}^{T} (p_t^s(q_{-it}^s, q_{it}^s) q_{it}^s - c_i(q_{it}^s)) - C_i q_i^{max} : 0 \leq q_{it}^s \leq q_i^{max}, \;\; t = 1, \ldots, T, s = 1, \ldots, S \Big\},$$

where C_i is an investment cost. As producers face much higher profit variations in the presence of renewables, we show how to replace the expectation-based objective with measure of risk, while maintaining tractability of the equilibrium problem.

Renewables and support schemes

The deployment of renewable energy is typically enforced by a target. For instance, the European Union requires at least 20% of its total energy consumption to be covered with renewable production by 2020. The market alone, however, cannot deliver the desired level of renewables in the EU. Renewable projects are often very capital intensive and can neither recover their investment costs nor compete with their conventional counterparts without public intervention. Means to incentivize investments comprise a variety of support schemes, the most widely implemented being a feed-in tariff (FIT, subsidy paid as a substitute for the electricity price), a feed-in premium (FIP, subsidy paid on top of the electricity price) and the trading of green certificates (TGC, certificate price received on top of the electricity price). Whereas tariffs are fixed by a regulator, electricity and certificate prices are determined by a market, and the support schemes therefore differ in producers' exposure to risk and ability to exercise market power.

The results from our stochastic equilibrium model show the following. Although FIT and FIP incentivise investment by reducing price risk, in the presence of a target, risk-averse producers are largely compensated for this risk exposure and may in fact obtain a higher expected profit with TGC. From a social point of view, tariffs have to be sufficiently high to incentivize investments that meet the target under any realization of future market conditions, whereas the ability of certificate prices to adapt to uncertainty results in maximum social welfare under perfect competition. In contrast, however, the exercise of market power may under certain circumstances distort the functioning of the market and change the socially optimal choice of support scheme.