My research: the world of extremes

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Most attention in probability theory has been devoted to the strong law of numbers (SLLN) and the central limit theorem (CLT). To some extent, the latter result explains the major role that the Gaussian distribution plays in science and applied fields. Both, the SLLN and the CLT, are results about *averages* of random quantities. Starting in the 1920s, various researchers discovered that extreme observations (such as the maximum or minimum in a sample of daily temperature or precipitation at a given site) are far from the Gaussian assumption. For example, hydrologists found the normal distribution unsuitable for describing high water levels of rivers and the sea. Motivated by these facts, applied mathematicians like von Mises, Gumbel, Fréchet, Fisher, Tippett,... laid the foundations of *extreme value theory* (EVT) and *extreme value statistics* in 1920–1960.

One of the triggers for the development of modern EVT was the tragic event of the flooding of parts of the Netherlands in 1953 in which more than 1800 people got killed. To avoid such events in the future a major problem was to determine a "reasonable" height of the dikes (see-walls) at the Dutch coast. The Dutch government required the dikes to be so high that the probability of a flood in a given year is less than 10^{-4} (a so-called one-in-10000 year event). This was a call for finding a suitable model for the distribution of extreme wave heights. In particular, suitable probabilistic and statistical techniques had to be developed for extrapolating into the tail of the distribution where no observations were available yet.

Classical EVT deals with the modeling of the extremes of independent data and their statistical analyses, in particular, the estimation of very high quantiles and far-out tails, such as the estimation of the height of the Dutch sea dikes. The determination of Value-at-Risk (VaR) in a financial institution is a closely related problem. VaR is an extreme low quantile, e.g. the 5%-, 3%- or 1%-quantile of the Profit-and-Loss distribution of a bank on a 10-day horizon. The goal is to use the VaR as a proxy to the capital reserve this bank has to keep in order to avoid ruin. In a sense, the estimation of the dike's height and of VaR are similar problems. The main difference is that financial data are typically serially dependent, in particalur, extremes tend to cluster in times of financial turmoil.

My research in the last 20 years has focused on the modeling and statistical analysis of extremal phenomena in discrete and continuous time.

My main focus has been on heavy-tail spatio-temporal phenomena, i.e., when the distribution of the underlying stochastic process lacks exponential moments and, in some cases, even certain power-moments. I have contributed to the theory of regularly varying stochastic processes, i.e., when their finite-dimensional distributions exhibit power-law tails. The 2016 Springer research monograph *Stochastic Models with Power-Law Tails. The Equation* $X = AX + B$, co-authored with Dariusz Buracweski and Ewa Damek – two harmonic analysts from the University of Wrocław, is outcome of my recent research. The fixed point equation in law $X = AX + B$ (for (A, B) independent of X) has fascinated me since the beginning of the 1990s when I realized that a major financial time series model, the ARCH, introduced by Robert Engle (1982) for the description of returns of speculative prices, actually satisfies the aforementioned fixed point equation. Jointly with Richard A. Davis (Columbia Statistics) and Bojan Basrak (a former PhD student, now at the University of Zagreb), we developed the probabilistic techniques for studying the statistical tools (such as the sample auto-correlation function) of these heavy-tailed time series.

One of my fields of interest is the study of the rare event probabilities $P(x_n^{-1} S_n \in A)$ where (S_n) is a random walk with dependent or independent heavy-tailed step sizes, A is a set bounded away from zero such that $x_n^{-1} S_n$ converges to zero in probability, hence $P(x_n^{-1} S_n \in A) \to 0$; I call them *heavy-tail large deviation probabilities.* Jointly with Olivier Wintenberger (Sorbonne) we derived precise bounds for large deviation probabilities of univariate and multivariate heavy-tailed time series. It turns out that large deviation probabilities are the right tools for studying the asymptotic behavior of the extreme eigenvalues in large random sample covariance matrices for heavy-tailed time series. This topic is related to recent joint work with Richard A. Davis and Johannes Heiny (a former PhD student at KU, now postdoc in Aarhus).