APPLIED PROBABILITY

MOGENS BLADT

My main areas of research are applied probability and statistical inference for Markov processes. In applied probability I have mainly been focusing on Markov processes and their applications to insurance risk, finance, and queueing theory. A larger proportion of my research has been dedicated to the theory and applications of so–called *phase–type* distributions and *matrix–exponential* methods.

To put this into a historical perspective, the exponential distribution has played a role similar to the normal distribution in statistics: it is a basic, nicely behaved and tractable distribution. Assuming that distributions are exponentials will often allow for deriving explicit solution in many stochastic models such as ruin probabilities in insurance risk or waiting time distributions in queueing systems. In 1909, A. K. Erlang, an outstanding Danish mathematician who had graduated from the University of Copenhagen in 1896 and been employed at the Copenhagen Telecom Company (KTAS), discovered that the exponential distribution provided a poor description for the duration of telephone conversations. Instead he considered the telephone conversations as going through a number stages, and suggested that it might be more adequate to view the duration of a conversation as the sum of a known number of independent exponentially distributed times. Such distributions, which is a special kind of Gamma distributions, are now known as (generalized) Erlang distributions.

Arne Jensen, an actuary and another employee at KTAS, continued the work of Erlang and came to the modern definition of a phase–type distribution as the time until absorption of Markov process with finite many states. It was published in his doctoral dissertation in 1954 and about a decade later he was named Professor of mathematics and statistics at Polytechnic School, now DTU, a position he held until 1990.

Phase–type distributions, defined in terms of absorption times, provides a class of distributions which may approximate any (positive) distribution arbitrarily close, and at the same time of provides explicit (exact) solutions to even complex stochastic models. Such solutions are often expressed in terms of matrix–exponentials or other functions of matrices.

There are several extensions of phase–type distribution like (1) matrix–exponential distributions, (2) Discrete phase-type distribution, (3) bilateral phase–type distributions, (4) multivariate phase–type distribution and (5) heavy tailed phase–type distributions. In (1) the aim is to reduce the dimension of the matrices involved, which can often be achieved by expressing the distributions in terms of matrices which are not generators of a Markov process, and which may be plugged into the same formulae as if they were generators.

While much research has already been carried out for classical basic phase–type distributions, I am now focusing on multivariate and heavy tailed distributions. While the latter class is important for describing data in insurance and finance (e.g. losses which usually have a tail different than exponentials), the multivariate distributions come in handy for constructing dependencies in models where it has been difficult to formulate a sensible and tractable sort of dependency. This is for example the case in insurance risk where dependencies between e.g. claim sizes and/or the times between them may be perceived or measured. Another area with a great potential for developing and applying multivariate phase–type distributions is for coalescent models in population genetics, where the presence of so–called recombinations can be described in terms of a multivariate phase–type distribution. Many of the different models can in this way be unified within the phase–type framework, offering general solutions instead of case-by-case solution to each model.

According to a famous proverb, I should tell you who my friends are so you can deduce who I am. Current work on discrete and mixed phase–type distributions is carried out with Bo Friis Nielsen, DTU, and students, heavy tailed phase–type theory Leonardo Rojas (Liverpool), Jorge Yslas (UCPH) and Hansjörg Albrecher (Lausanne) while applications of multivariate phase–type distributions in genetics is joint with Asger Hoobolth (Aarhus) and Arno Siri-Jégousse (Mexico City). Somewhat unrelated to phase–type theory, I am also working on both probabilistic and statistical aspects of discretely observed Markov processes (Markov jump process or (multivariate) diffusions) jointly with Michael Sørensen (UCPH) and Fernando Baltazar (Mexico City).