Special FX

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Much of my recent research has been on special events (or cases) in foreign exchange (FX) markets. Rather than talk poetically at length about the importance of FX markets and normative vs. positive approaches to finance, let me simply present two cases: The Swiss Floor and Brexit/Trump.

The Swiss Floor.[1] The eurozone debt crisis, which started in 2009, lead to a considerable strengthening of the Swiss franc relative to the euro; Switzerland was seen as a safe haven. Throughout the 00'ies one euro would buy you about one-and-a-half Swiss francs, but from 2009 that number started dropping. While such a strengthening is good for national pride, it is not good for the sectors of the economy that sell goods and services to the eurozone; export and tourism. In the summer of 2011 when the exchange rate approached parity, the situation became unbearable for the Swiss National Bank (SNB). On September 6, 2011 they instigated a so-called floor on the Swiss franc – euro exchange rate: A guarantee that 1 euro would always buy you at least 1.20 Swiss francs.

Two questions then arise: How credible is the SNB guarantee? Where would the exchange rate be without the guarantee? To answer, we view the observed exchange rate at time-point t in the guarantee period, $S(t)$, as the sum of a latent exchange rate, $V(t)$ — the *true* exchange rate, or where the exchange rate would be without the guarantee — and the value of the guarantee, which can be modelled as a that of strike-1.20 American put option on the latent exchange rate; you can sell your 1 euro for 1.20 Swiss francs to the SNB any time you like.

We now assume that V follows a geometric Brownian motion with volatility σ. Because both the Swiss and the eurozone interest rates were effectively zero during the guarantee period, American optionality adds nothing over European optionality, and we can write

$$S(t) = V(t) + \mathrm{B/S}^{put}(V(t)|\sigma, \tau), \tag{1}$$

where $\mathrm{B/S}^{put}$ denotes the Black-Scholes put option formula (which in the realm of FX modelling is often called the Garman-Kohlhagen formula) and we treat the time to expiry

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[1] Hanke, Poulsen & Weissensteiner (2017a)

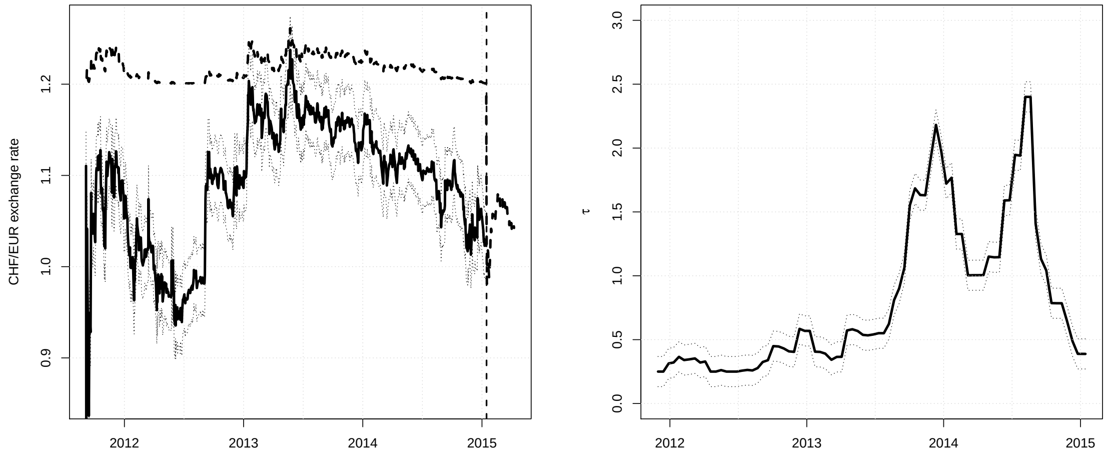

Figure 1: Left hand panel: Estimated latent (fully drawn with grey confidence bands) and observed (dashed) Swiss franc–euro exchange rate. Right hand panel: Estimated gaurantee length (τ; measured in years).

of the put option as a parameter reflecting the market's view on the credibility (i.e. the length) of the SNB guarantee. Equation (1) can – after applying a Jacobian correction term, that can be found in closed form – be used on a time series of observed exchange rates to obtain maximum likelihood estimates of the parameters σ and τ and of the latent exchange rate.

Being able to estimate the model, how do we validate (a term I use deliberately loosely) it? There are sound academic ways to do that; these take up quite a bit of the paper. However, on January 15, 2015 the SNB removed the guarantee thus providing us with one very hard data point on where the exchange rate would be without the guarantee. The left hand panel of Figure 1 shows that the model provided quite an accurate estimate of the realized exchange rate after the removal of the guarantee. The January 14 estimate for the latent exchange rate was 1.023 with a 95% confidence interval of [0.986,1.060]. The observed exchange rate on January 15 was 1.0508, and it dropped further on January 16, hovering between 0.981 and 1.016 over the following days. While the guarantee removal was inherently unpredictable (like a jump in a Poisson process), the right hand panel of Figure 1 shows that the market's perception of the credibility of the guarantee dropped sharply in months leading up to the removal.

Brexit/Trump.[2] On June 23, 2016 the United Kingdom held a referendum on whether to leave the European Union ('Leave' or 'Brexit') or not ('Remain'). 51.9%

[2] Hanke, Poulsen & Weissensteiner (2017b)

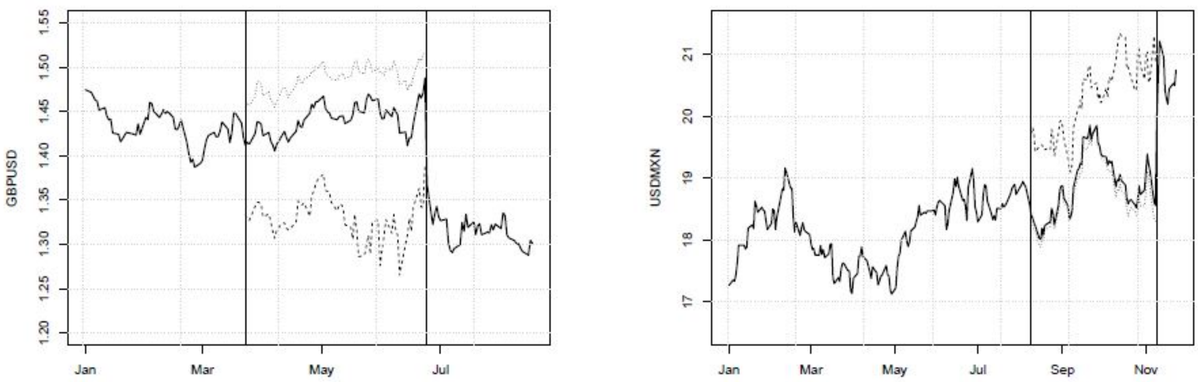

Figure 2: Left hand panel: US dollar – British pound exchange rates leading up to the EU referendum. The fully drawn curve is the observed exchange rate; the dashed (resp. dotted) curve is the time t-expected post-referendum exchange rate conditional on 'Leave' ('Remain'). Righ hand panel: Mexican peso – US dollar exchange rates leading up to the presidential election. The fully drawn curve is the observed exchange rate; the dashed (resp. dotted – barely visible under the fully drawn curve) curve is the time t-expected post-referendum exchange rate conditional on Trump winning (not winning).

of the votes cast were for leaving.

We may ask: Which probabilities did the market attach to the two outcomes at various points prior to the referendum? What was the market's view of the *impact* of either of the two outcomes? What was expected to happen *if* (or: conditional on) the outcome was, say, Leave? The first question is easy, as sizeable betting markets for the direct outcome existed. (Technically, these give us so-called risk-neutral probabilities, but ...) The second question requires richer data and modelling — and that we take impact to mean impact on the US dollar – British pound exchange rate. We attack it thus:

- On each day prior to the referendum market we use observed prices of FX put and call options to estimate the risk-neutral density of the exchange rate shortly after the referendum; up to discounting, it's the second derivative wrt. strike.
- We then fit this density as best we can as a mixture of two lognormals, where the mixing parameter is given by betting market probabilities and risk-neutrality basically removes one further parameter.

This allows us to estimate the market's exchange rate expectation conditional on either outcome. This is shown in the left hand panel of Figure 2. Even though the market viewed Leave as the less likely outcome (probability about 20%),[3] it shows that the

[3] Some say "the market got the Leave probability wrong". That's a very bold statement to make based on just one observation. It's akin to throwing a die an claiming it can't be fair because a six come up. Although, arguably, if you throw in Trump's victory also, you may see a pattern emerge ...

market was highly accurate in its prediction of what whould happen to the exchange rate if Leave were the outcome. Further analysis reveals that the observed probabilities are only weakly correlated the estimated conditional expected values. So there is separation bewteen likelihood and impact; something that for instance credit risk models struggle to achieve.

A similar analysis can run be in connection to the 2016 US presidential election with the event being 'Trump wins' and the impact being on the US dollar – Mexican peso exchange rate. (Trump made – shall we say – strong remarks directed at Mexico.) The conclusion (right hand panel of Figure 2) is the same: Not the most likely outcome (also about 20%); consistent market reaction.

Epilogue. The Devil's Advocate would say that these are 20/20 hindsight models that conviniently justify or rationalize market behaviour in spite of secretive, unpredictable, or irrational agents (the SNB and the common voter). To this I would say that a key part of the empirical set-up is that model based forecasts at time t are based only on the information that market participants would actually have at time t. The Advocate might counter that while this removes first order in-sample bias, there is still the subtle issue that in reality you the modeller chooses the model only after seeing the full data sample. This is indeed a serious issue with any empirical analysis in economics, but there are two reasons I don't lose sleep at night over it: (a) We use causal models rather than purely phenomenolocical ones (Lucas critique), (b) We have a 'paper-trail documentation' that we did propose the models before the full data sets were known (before the removal of the guarantee, before the US election).