My research

by *Christian Furrer*, January 28, 2022

I am interested in multi-state modeling in the mathematics of life insurance with a view towards applications in health and disability insurance as well as pensions. The starting point for my research is the classic Markov chain model, introduced in the late sixties by Jan M. Hoem, who would later take up a professorship at the Laboratory of Actuarial Mathematics in Copenhagen. In the classic Markov chain model, the process Z governing the development of the insurance contract is assumed to be a Markov chain with transition rates μ. Furthermore, the contractual payments B are required to be of the form

$$B(\mathrm{d}t) = \sum_i \mathbb{1}_{\{Z_{t-}=i\}} B_i(\mathrm{d}t) + \sum_{i,j:i\neq j} b_{ij}(t) N_{ij}(\mathrm{d}t)$$

for deterministic functions B_i and b_{ij} describing sojourn and transition payments, respectively. Here N is the multivariate counting process associated with Z, so that N_{ij} counts the number of transitions from state i to state j. Thiele's differential equations, which describe the dynamics of so-called state-wise prospective reserves, play a central role in life insurance mathematics. Besides providing efficient computational schemes for the insurer's balance sheet, characterizing the dynamics of prospective reserves is for example crucial for the study of the emergence and decomposition of surplus. If we presume a maximal contract time n and a deterministic short rate r, and if we let $(V_i)_i$ denote the state-wise prospective reserves given by

$$V_i(t) = \mathbb{E}\left[\int_t^n e^{-\int_t^s r(u)\,\mathrm{d}u} B(\mathrm{d}s) \,\middle|\, Z_t = i\right],$$

then Thiele's differential equations read

$$V_i(\mathrm{d}t) = V_i(t)\, r(t)\,\mathrm{d}t - B_i(\mathrm{d}t) - \sum_{j:j\neq i} \big(b_{ij}(t) + V_j(t) - V_i(t)\big)\mu_{ij}(t)\,\mathrm{d}t, \qquad V(n) = 0. \tag{1}$$

A significant proportion of my research aims at moving 'beyond' the classic Markov chain model to offer insights that are not tied to specific assumptions on for example the intertemporal dependence structure of the process governing the development of the insurance contract. Essentially, any multi-state model consists of three components: a probabilistic model, a specification of sojourn and transition payments, and potential restrictions on the available information. In case of a Markov chain model, the probabilistic model is a Markov chain, the sojourn and transition payments are required to be deterministic, and the available information consists of the current state of the insurance contract.

I am particularly interested in complications arising from the inclusion of policyholder behavior, which leads to so-called scaled payments, and non-monotone information. Non-monotone information structures appear in various contexts, including

- Legal constraints: By exercising the 'right to be forgotten' according to Article 17 of the General Data Protection Regulation of the European Union, the policyholder may ask the insurer to delete health related data at discretion.

- Big data: Data from for instance activity trackers may improve forecasts of mortality and morbidity of individual insured, but data providers often implement auto-deletion mechanisms to improve data privacy.

Together with Marcus C. Christiansen from the University of Oldenburg, I am studying the dynamics of state-wise prospective reserves in the presence of non-monotone information. In case of monotone information, the dynamics of state-wise prospective reserves are closely related to the martingale representation theorem. This is not the case under information discarding, which has lead us to conduct research on the interface between actuarial mathematics and probability theory. The current highlight of our work is a generalized stochastic Thiele equation that allows for information discarding. The result is quite involved and better left unstated for now, but it bears a close resemblance to Thiele's differential equations. To illuminate the latter fact, note that the terms

$$\big(b_{ij}(t) + V_j(t) - V_i(t)\big)\mu_{ij}(t)\,\mathrm{d}t$$

from (1) concern the arrival of new information at time t. They comprise the change in reserve upon a transition of Z from state i to state j at time t, namely $b_{ij}(t) + V_j(t) - V_i(t)$, times the infinitesimal probability of this transition, namely $\mu_{ij}(t)\,\mathrm{d}t$. In the generalized stochastic Thiele equation, additional yet similar terms related to information discarding make an appearance.

Before I joined the department as a tenure track assistant professor in late 2020, I was an Industrial PhD student in a collaboration between the department and PFA Pension. It is my aim to build bridges between academia and practice, an effort for which I also have been awarded the 2019 Gauss Prize. Currently, I am engaged in the project framework InterAct which launched January 1 and is a collaboration between the department and eight of the country's insurance companies and pension funds. The vision for InterAct is to elevate the department's research and education activities in actuarial mathematics *with and towards* the industry by creating significant increases in volume, variation, and practical relevance to the benefit of all parties.