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Preprint
May
1992

3

Institute of Mathematical Statistics
University of Copenhagen

ISSN 0902-8846

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1. Introduction

Most macroeconomic time–series behave in a nonstationary manner. That this has crucial implications for econometric modeling has, however, only recently become gradually understood. While the nonstationarity of the data was previously considered a nuisance and therefore largely ignored, recent experience with cointegration analysis has pointed to its great potential as a statistical means to distinguish between long–run relations and short–run dynamic adjustment. The statistical classification into stationary and nonstationary components of a time–series process has provided a natural framework for the analysis of economic concepts such as long–run equilibria, or long–run steady–states and short–run dynamic adjustment towards these. The possibility to ask interesting questions within a well–defined statistical model such that they can be validly tested makes this approach potentially useful for economic investigation. The usefulness is, however, closely related to whether one can identify within the statistical model economic structures of interest. The treatment of identification in a statistically well–specified model when the data are nonstationary is the issue of this paper.

When the empirical model is estimated with data that are nonstationary in levels we will have to discuss identification of the short–run structure, i.e. identification of the equations, as well as identification of the long–run structure, i.e. identification of the cointegration relations. It appears therefore that we have the possibility of two types of structural equations: Structural relations between the levels, the long–run or cointegrating relations, and structural equations for the changes of the process which also involve the disequilibrium error from the long–run relations. A set of relations <u>between</u> variables becomes a set of equations <u>for</u> a subset of the variables if the relations are solved for, or normalized on, some subset of the variables. We will discuss identification w.r.t. the long–run relations, the long–run structure, and w.r.t. the short–run adjustment, the short–run structure. It appears that for a full understanding of identification it is useful to distinguish between identification in three different meanings:

1) formal identification, which is related to a statistical model
2) empirical identification, which is related to the actual estimated parameter values, and
3) economic identification, which is related to the economic interpretability of the estimated coefficients of a formally and empirically identified model.

For identification to be of practical interest the conditions for identification in all three cases have to be satisfied relative to the empirical problem, which as a crucial part involves the choice of data.

The organization of this paper is the following: In section 2 the empirical problem and the interesting hypotheses are discussed in a general framework given by the ISLM model and the buffer stock theories of monetary disequilibrium. In section 3 a statistical model for first order integrated data is suggested and represented in the reduced form and the structural form respectively. The parameters of the model are partitioned into the short–run and the long–run parameters and it is shown that the analysis of the long–run structure can be performed in either representation. In section 4 the identification problem is discussed in terms of structural hypotheses on the long–run and the short–run structure. A general result for formal identification in a statistical model is given, and empirical identification is given a precise definition. A switching algorithm for calculating the restricted eigenvector is proposed. The concepts are illustrated with an empirical analysis of Australian monetary data consisting of money stock, real income, prices and two interest rates. In section 5 the analysis of identification in the long–run structure is performed. The uniqueness of the unrestricted cointegration space is discussed followed by two cases of exact identification using identifying restrictions. Lack of formal identification is illustrated and several hypotheses of interest are tested until the final long–run structure is arrived at. This is shown to be formally, empirically and economically identified. In section 6 the unrestricted short–run structure of the model is examined using three different unrestricted parameterizations of the statistical model. Just identification is achieved by imposing p–1 zero restrictions on each equation and the rank conditions for formal identification are shown to be satisfied. However, it is then demonstrated that the estimated model shows lack of empirical identification. This is related to the fact that the chosen information set is not sufficient large to discriminate between all five equations. The final model consists of three simultaneous equations for short interest rate, real income and nominal money stock, whereas the long bond rate and the price inflation are given in reduced form, such that identification of these two equations relative to the others is achieved by restricting the error covariance matrix instead of the structural coefficients.

2. The empirical background.

The empirical problem of this paper can be seen as a direct continuation of the analysis in Juselius (1991a). The reason for the empirical investigation was previous shortcomings in the search for a stable money relation in Australia. See for instance Milbourne (1990) and Stevens et.al (1987). The aim of the empirical analysis in Juselius (1991a) was twofold; i) to investigate whether it was possible to find a stable money relation using the multivariate cointegration analysis, ii) to investigate whether the monetarist assumption that excess money causes inflation can be directly or indirectly verified.

As discussed in the above mentioned paper, the lack of close correspondence between the theoretical variables and the observational variables made it difficult a priori to know which definition of money stock was the most appropriate for a monetary relation. This was the reason for choosing alternative empirical measurements to represent the theoretical variables. The long–run analyses were based on the vector $z_t = (m, y, p_y, i_s, i_b)$ for two different measurements of money, M1 and M3, and two different measurements of income, GNE and GDP, the implicit price index and two interest rates; the 3 month commercial bill rate and the 10 years bond rate.

But instead of the conventional LM relation, which was the starting point of the analysis, the analysis of the statistical model provided empirical evidence about the interaction of the real and monetary sectors of the economy and thus suggested the full ISLM model as a more appropriate economic framework. With the combination m = M1, y = GNE essentially two long–run relations were found that seemed primarily to describe the private sector transactions demand for money and an a real domestic demand relation, whereas with the combination m = M3 and y = GDP three long–run relations were found; a domestic demand relation and two interest rate relations, one relating the short interest rate to the long bond rate and the other describing a real long–term bond rate relation. These findings were quite interesting, since they seemed to relate directly to many of the issues in the current discussion about monetary transmission mechanisms.

An important limitation of the conventional ISLM model is that it is entirely concerned with the analysis of comparative–static equilibrium positions. When there is a change in one of the exogenous variables the endogenous variables adjusts instantaneously to the new equilibrium position, albeit via an adjustment path that also may involve the other variables. This is, however, a theoretical simplification that does not correspond to the actual behaviour of the observed variables. Since in reality the economy is constantly experiencing random shocks, one can say that it is virtually by definition always out of static equilibrium. Nevertheless the static comparative solutions of the conventional ISLM model can be considered a useful expositional device for the analysis of the long–run structure of a dynamic model.

The theoretical assumptions of the ISLM model are fairly standard but, depending on whether prices are assumed fixed or flexible, it may have a Keynesian or a neoclassical interpretation (see for instance Levacic and Rebman (1989)). The Keynesian version of the model treats output and the interest rate as endogenous variables assuming the price level fixed until full employment is reached, whereas the neoclassical version of the model treats the price level and the interest rate as endogenous variables. We will use this framework to see if we can empirically distinguish between these two versions, or whether the data seem to support one version more than the other. In the simplest form of the ISLM model we have:

$$y = f(\underset{-}{i}),\ \ p = f(\underset{+}{m-y}),\ i = f(\underset{+}{\Delta p}),\ m^d = f(\underset{+\ -}{y,i}) \text{ and } m^s = m^d$$

where m^s is the supply of money assumed exogenously given, m^d is the demand for money, and f(x) is a linear function of x.

For the short–run structure of the model we need a theoretical framework that allows for dynamic adjustment both to the changes of the determinants and to the long–run steady–states. The monetary disequilibrium buffer stock theories seem to come closest in offering a theoretical framework for the dynamic short–run analysis. They are developed for an economy with disequilibrium primarily in the the monetary sector, though some of the models also allow for disequilibrium in the real sector. Milbourne (1988) gives a useful review of these models, suggesting a classification into the following four groups: i) models assuming real flow equilibrium and money stock disequilibrium ii) "Keynesian" flow disequilibrium and stock disequilibrium, iii) flow–disequilibrium and rational expectations and iv) models with endogenous money.

In the last group, money stock is treated as an endogenous variable and therefore explicitly or implicitly it is assumed to be primarily demand determined. Empirical applications of these models typically add some additional explanatory variables, often representing supply shocks, to improve the fit. As mentioned above we found in Juselius (1991a) that M1 seemed to be primarily demand determined, whereas M3 seemed to be exogenous for the long–run parameters and therefore more likely to be supply determined. Therefore the theoretical considerations from the fourth group of models seem a priori more relevant for the analysis of M1. The third group is based on the assumption that only unanticipated changes in the money stock can cause a monetary disequilibrium. An important aspect of these models is the assumption of price flexibility.

The second group relaxes the static assumptions of the ISLM model by allowing for temporal disequilibria both in the monetary and the real sector, whereas the first group assumes that the real sector of the economy is always in temporary equilibrium. The consequence of a money supply shock is that agents adjust their expenditure accordingly and ends up at a new temporary equilibrium, which however might not correspond to the neoclassical equilibrium. The difference between these two versions does not seem to be essential in our model framework, since we would generally consider the deviation between a temporary equilibrium and the long–run steady–state (neoclassical equilibrium) as a deviation from the long–run steady–state. The first and the second group of buffer stock models provide a theoretical framework that seems to be consistent with the statistical models of the data generating process (DGP) for the case M3 with GDP that were estimated in Juselius (1991a).

In this review we have deliberately considered a broad range of possible economic models instead of restricting us to one specific. Our approach is to start with a well–defined statistical model, within which we will be able to ask not just a restricted number of hypotheses relevant for one economic model, but as many as seem relevant given the basic features of the DGP. For instance we will not test specific hypotheses based on the assumption of instantaneous market clearing if the statistical model suggest adjustment behaviour. Thus our modeling approach is data based in the sense that we look at the data as structured by the statistical model through the different glasses of a variety of economic theories or hypotheses.

Some of the more important prior hypotheses we are going to investigate empirically are given below:

Does money matter for real aggregate income? Is it possible by expansion of money supply in excess of the real productive level in the economy to permanently increase real income or is the final effect only an increase in the inflation rate? Should money supply or the interest rates be used as monetary instruments? Is money stock endogenously or exogenously determined? What is the difference in this respect between the narrow and broad definition of money stock? Can we estimate a stable aggregate money relation for Australia?

Is the interest rate determined in the money market or outside? Does it respond to excess demand in the goods market or alternatively to excess demand in the money market? Do the interest rates adjust quickly or slowly? Is the price level adjusting to excess demand in the money market or to excess demand in the goods market?

3. A statistical model for the DGP.

Here we will continue the analysis of the case M3 with GDP for which the data vector z_t is given by:

$$z_t = [m3, y, p, i_s, i_b]_t' \qquad t = 1975{:}3,\ldots,1991{:}1$$

where m3 is the log of broad money stock in nominal terms, y is the log of the real GDP, p is the log of the implicit price deflator of GDP, i_s is the three month commercial bill rate, i_b is the 10 year bond rate. In addition we needed a vector of dummy variables given by:

$$D_t = [\ s1, s2, s3, 1, t, D84\ ] \qquad t = 1975{:}3,\ldots,1991{:}1$$

s1, s2 and s3 are centered seasonal dummies, and D84 is a shift dummy variable with value 1 for t = 1984:1,...,1991:1, 0 otherwise. The latter was needed to account for the effects of the float of the dollar and the abolishment of most capital controls in the banking sector. The choice of the variables are discussed in more detail in Juselius (1991a).

A priori all of the variables in z_t must be assumed stochastic and therefore should be statistically modeled. We define $Z_t = [z_1,\ldots,z_t]$. The probability formulation for the whole data set can be given in terms of sequential conditional probabilities:

$$P(Z_t \mid Z_0, D_t;\theta) = \prod_{t=1}^{T} P(z_t \mid Z_{t-1}, Z_0, D_t;\theta) \qquad (1)$$

where $Z_0 = [z_0, z_{-1},\ldots]$ is a matrix of initial values. By assuming multivariate normality for the conditional process, the mean of the conditional process is linear in the parameters and the reduced form model (2) can be regarded as a statistical model for the data:

$$z_t = \Pi_1 z_{t-1} + \dots + \Pi_k z_{t-k} + \psi D_t + \epsilon_t \tag{2}$$

$$\epsilon_t \sim \text{Niid}_p(0,\Sigma) \tag{3}$$

where it is assumed that $\Pi_{k+i} \simeq 0$ for $i = 1, 2,\dots$. The assumptions of normality, independence, variance homogeneity and truncation are all testable and should indeed always be tested before accepting (2) as a satisfactory statistical model for the DGP. For further discussion of the probability approach to econometrics see for instance Haavelmo (1944), Hendry & Richard (1983). The reduced form model (2) can be reformulated as:

$$\Delta z_t = \Gamma_1 z\Delta_{t-1} + \dots + \Gamma_{k-1}\Delta z_{t-k+1} + \Pi z_{t-1} + \psi D_t + \epsilon_t \tag{4}$$

where $\Gamma_j = -\sum_{i=j+1}^{k} \Pi_i$, and $\Pi = I - (\Pi_1 + \dots + \Pi_k)$. If data are nonstationary, say z_t is integrated of order one, hereafter I(1), then the matrix Π has to be of reduced rank:

$$\Pi = \alpha\beta' \tag{5}$$

where α and β are pxr matrices and $r < p$. For the statistical analysis of this model, see Johansen (1988) and Johansen and Juselius (1990). The reduced form model (4) with cointegration (5) is now given by (6). [2]

$$\Delta z_t = \Gamma_1 z\Delta_{t-1} + \alpha\beta' z_{t-1} + \psi D_t + \epsilon_t \tag{6}$$

where $\beta' z_t$ are the so called cointegrating relations. The reduced form as given by (6) and (3) uniquely defines the probability distribution of the data and in that sense qualifies as a statistical model for the data. It is useful for the subsequent discussion to partition the parameters of (6) and (3), $\theta = \{\Gamma_1, \psi, \Sigma, \alpha, \beta\}$, into the set of short–run parameters $\theta_s = \{\Gamma_1, \psi, \Sigma, \alpha, \}$ and the set of long–run parameters $\theta_l = \{\beta\}$. A structural model is defined by the economic formulation of the problem and can for instance be given by:

$$A_0 \Delta z_t = A_1 \Delta z_{t-1} + a\beta' z_{t-1} + \tilde{\psi} D_t + u_t \tag{7}$$

where $u_t \sim \text{Niid}_p(0,\Omega)$ and the structural form parameters $\lambda = \{A_0, A_1, a, \tilde{\psi}, \beta, \Omega\}$ are usually restricted. As with the reduced form parameters λ is partitioned into the set of structural short–run parameters $\lambda_s = \{A_0, A_1, a, \tilde{\psi}, \Omega\}$ and long–run parameters $\lambda_l = \{\beta\}$. The relation between θ_s and λ_s is given by:

$\Gamma_1 = A_0^{-1} A_1$, $\alpha = A_0^{-1} a$, $\epsilon_t = A_0^{-1} u_t$, $\psi = A_0^{-1}\tilde{\psi}$ and $\Omega = A_0^{-1} \Sigma A_0'^{-1}$.

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[2] For notational simplicity we will from now on assume that the lag length $k = 2$, since $k \geq 2$ is sufficient to allow us to distinguish between the short–run structure and the long–run structure of the model.

4. Identification when data are nonstationary

For a unique identification at least p(p–1) restrictions have to be imposed on the short–run structural form parameters λ_s. However, the long–run parameters are the same both in the reduced form and the structural form implying that the identification of the long–run structure can be done in either form. This will be shown to be of crucial importance since it implies that identification of the long–run structure can be done in the reduced form model which is statistically well determined.

Below we will discuss identification separately for the long–run and the short–run structure and consequently specify

(1) Structural hypotheses on the long–run structure (β)

(2) Structural hypotheses on the short–run structure (A_0, A_1, a, $\tilde{\psi}$, Ω)

Macroeconomic theory is usually quite informative about long–run effects thereby suggesting economically well founded identifying restrictions on the long–run structure, whereas much less is often known about the short–run structure. It will be discussed below that, given an identified long–run structure, identification of the short–run structure can often be achieved by means of identifying restrictions on the weight parameters, or alternatively given identifying restrictions on the weight coefficients a, identification of the long–run structure can be achieved. This suggests that the identification of the long–run structure is closely related to the identification of the short–run structure and vice versa through the weight coefficients a.

4.1. The identification of the long–run structure.

In order to identify the long–run relations we formulate restrictions on the individual relations. We let R_i be $p \times k_i$ matrices of full rank and let $H_i = R_{i\perp}$ be $p \times s_i$, ($k_i + s_i = p$) such that H_i is of full rank and satisfies $R_i' H_i = 0$. Thus there are k_i restrictions and s_i parameters to be estimated in the i′th relation. The cointegrating relations are thus assumed to satisfy the restrictions $R_i'\beta_i = 0$, or equivalently $\beta_i = H_i\varphi_i$ for some s_i–vector φ_i, that is,

$$\beta = (H_1\varphi_1,...,H_r\varphi_r), \tag{8}$$

where the matrices $H_1,...,H_r$ express the economic hypotheses to be tested against the data. See Table 2–5 for examples of choices of such matrices. We now want to express that the model defined by (8) identifies the cointegrating relations. The well known rank condition expresses that the first equation, say, is identified if

$$\text{rank}(R_1'\beta_1,...,R_1'\beta_r) = \text{rank}(R_1'H_1\varphi_1,...,R_1'H_r\varphi_r) = r-1, \tag{9}$$

such that no linear combination of $\beta_2,...,\beta_r$ can produce a vector that "looks like" the coefficients of the first relation, that is, satisfies the restrictions that is defining the first relation. In Johansen (1992b) a condition is given for a set of restrictions to be identifying, in the sense that a set of β–vectors exists which satisfies the restrictions and such that the rank condition is satisfied.

THEOREM 1. *The set of restrictions is formally identifying if the following condition is satisfied: For all i and k = 1,...,r−1 and any set of indices* $1 \leq i_1 < \ldots < i_k \leq r$ *not containing i it holds that*

$$rank(R_i'H_{i_1},\ldots,R_i'H_{i_k}) \geq k. \tag{10}$$

As an example consider r = 2, where the conditions reduce to the conditions

$$r_{i.j} = \text{rank}(R_i'H_j) \geq 1, \; i \neq j.$$

If r = 3 the conditions to be checked are

$$r_{i.j} = \text{rank}(R_i'H_j) \geq 1, \; i \neq j,$$

$$r_{i.jm} = \text{rank}(R_i'(H_j,H_m)) \geq 2, \; i,j,m \text{ different.}$$

In the empirical applications below we check the rank condition using the identity:

$$r_{i.jm} = \text{rank}(R_i'(H_j,H_m)) = \text{rank}(H_j,H_m)'(I-H_i(H_i'H_i)^{-1}H_i')(H_j,H_m)$$

which can be determined by finding the eigenvalues of symmetric matrices.

In checking the identification condition (9) it is required to know the parameter value. Thus in practice one often checks the rank condition for "generic" coefficients, and if the the rank condition is satisfied then one concludes that the equations are identified. The theorem gives an easy algebraic condition to check formal identification, thus avoiding the generic coefficients, or the lack of some "freakish conjunction of coefficients", (Johnston 1984, p.455), and allows one to check explicitly if the restrictions are identifying. Thus the usual rank condition (9) requires the knowledge of the true parameters, whereas condition (10) is a property of the statistical model or the parameter space which defines the model.

DEFINITION 1. *For identifying restrictions it holds that* $rank(R_i) \geq r-1$, *and if equality holds the i′th relation is exactly identified, if inequality holds then the i′th relation is overidentified. The system is exactly identified if* $rank(R_i) = r-1$ *for all i, and overidentified if it is identified and rank* $R_i > r-1$ *for at least one i.*

Once the system has been checked to be identified it is possible to test further overidentfying restrictions on the parameters by restricting the variation of the parameters, that is, by decreasing the spaces H_i, defining the variation. The likelihood ratio statistics derived for such hypotheses are asymptotically χ^2 distributed such that usual inference can be performed.

Note that when accepting further overidentifying restrictions the rank conditions (10) need not be satisfied any more and we can meet the situation that the estimated parameters indicate that a further reduction in the statistical model is possible. However, when we are imposing further restrictions on a formally identified model we can get the result that the new model is no longer identified. This is related to the discussion of the generic parameter values above. If for instance the rank condition (10) is satisfied under the condition that a certain coefficient is nonzero, then it is easy to assume that generically such a condition is satisfied. But it may of course be the case that the actual or true value of the parameter is in fact zero, or equivalently that the estimate is not significantly different from zero. In this case we say that even though the statistical model is formally identifying, the economic model is not empirically identified. We formalize these considerations:

DEFINITION 2. *An economic model specified by the parameter value ϑ, say, is formally identified if ϑ is contained in the parameter space specified by identifying restrictions. It is empirically identified if ϑ is not contained in any nonidentified sub model.*

In a formally identified model the parameters can be estimated under the restrictions by an iterative procedure which involves successive reduced rank regressions. A similar procedure has been suggested in Johansen and Juselius (1992) for restrictions on one vector only. In order to see the principle behind the the switching algorithm we take the error correction term from equation (6) and write it as

$$\alpha\beta' z_{t-1} = \alpha_1\beta_1' z_{t-1} + \ldots + \alpha_r\beta_r' z_{t-1} = \alpha_1\varphi_1' H_1' z_{t-1} + \ldots + \alpha_r\varphi_r' H_r' z_{t-1}.$$

It is seen that for fixed values of $\varphi_2,\ldots,\varphi_r$, or $\beta_2,\ldots,\beta_r$, the statistical calculations derived from (6) are performed by reduced rank regression of Δz_t on $H_1' z_{t-1}$ corrected for all the stationary and deterministic terms, that is, $\beta_2' z_{t-1},\ldots,\beta_r' z_{t-1}, \Delta z_{t-1}$ and D_t. This determines the estimate of φ_1 and $\beta_1 = H_1\varphi_1$. Next fix the values $\beta_1,\beta_3,\ldots,\beta_r$ and perform a reduced rank regression of Δz_t on $H_2' z_{t-1}$ corrected for all stationary and deterministic terms. This determines β_2. By continuing the algorithm until convergence we determine the maximum likelihood estimator for the cointegrating relations under the restrictions given by (8).

As initial values one can choose the unrestricted estimates, $\hat{\beta}$, but that is in general not the best choice, especially since the ordering given by the unrestricted eigenvectors need not correspond to the ordering given by $H_1,\ldots,H_r$. Thus it is preferable to use as a starting value for β_1, say, the linear combination of the unrestricted estimates which is closest to $sp(H_1)$. This is found by solving the eigenvalue problem:

$$|\rho\hat{\beta}'\hat{\beta} - \hat{\beta}' H_1(H_1' H_1)^{-1} H_1'\hat{\beta}| = 0,$$

for the r eigenvalues $\rho_1 > ... > \rho_r$ and $v_1,...,v_r$, and choose as initial value for β_1 the vector $\hat{\beta}v_1$. This choice has the extra advantage that for exactly identified equations no iterations are needed.

4.2. Identification of the short–run structure.

A similar formulation is possible for the identification of the short–run dynamics. We now assume that the cointegrating relations have been properly identified and estimated, and want to proceed to identify the short–run dynamics for fixed values of the long–run coefficients. In the structural error correction model (7) we define the parameters $A' = (A_0, -A_1, -a)$ and the stationary process $X_t' = (\Delta z_t', \Delta z_{t-1}', z_{t-1}'\beta)$. The model then becomes

$$A'X_t = \psi D_t + u_t.$$

We next formulate identifying restrictions on the columns of $A = (A_1,...,A_p)$, that is, we assume that

$$A = (H_1\varphi_1,...,H_p\varphi_p).$$

Again one can check the conditions given in the Theorem to see if the restrictions defining the model are identifying, and when this is done the estimation can be performed by the eigenvalue routine described before. To see this note that the likelihood function has the form

$$\log L(A,\Omega,\psi) = -\tfrac{1}{2}T\log|\Omega| + \tfrac{1}{2}T\log|A_0A_0'| -\tfrac{1}{2}\Sigma_1^T(A'X_t - \psi D_t)'\Omega^{-1}(A'X_t - \psi D_t).$$

Maximization with respect to ψ and Ω shows that

$$\log L_{max}(A) = \tfrac{1}{2}T(\log|A'MA| - \log|A'S_{xx}A|),$$

where S_{xx} is the product moment matrix of X_t corrected for the deterministic terms, and the matrix M is defined by

$$M = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$

so that $|A'MA| = |A_0A_0'|$. This maximization problem has the same structure as the one obtained for the estimation of the cointegrating relations and the same algorithm can be applied, see Johansen (1992b).

5. Identification of the long–run structure [3]

In this paper we have chosen to investigate the M3 & GDP case of the empirical applications treated in Juselius (1991a) within the reformulated framework discussed in

[3] The calculations have been performed with the computer package CATS in RATS, Juselius (1991c).

section 3. It was found that an unrestricted VAR–model of order two provided a satisfactory description of the covariance structure of the data. The likelihood ratio unit root test indicated that the rank of the Π–matrix was equal to 3. Thus a statistically well–defined empirical model describing the covariance structure of the data can be given by:

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + \alpha\beta'\tilde{z}_{t-1} + \psi D_t + \epsilon_t.$$
$$\epsilon_t \sim \mathrm{niid}_p(0,\Sigma)$$

where $\tilde{z}_t = (m3, y, p, i_s, i_b, t)$, α is of dimension 5x3 and β are of dimension 6x3, suggesting that there are three linearly independent stationary long–run relations in this data set. As discussed in section 4 the long–run structure β is identical in the reduced and the structural form and identification of the long–run relations can therefore be done in model (6). In section 5.1. below we will address the question whether it is possible or reasonable to give this particular parameterization of the covariance structure of the data an interpretation in terms of an underlying economic structure. We will then discuss ways of simplifying this structure by imposing data consistent restrictions and finally use the procedure developed in section 4 to investigate whether it is possible to impose such restrictions that can identify interpretable economic relations.

5.1. The uniqueness of the unrestricted estimates of β.

The unrestricted estimates of α and β are calculated given the following conditions

(c.i) Stationarity, i.e. $\hat{\beta}' z_t \sim I(0)$

(c.ii) Conditional independence of $\hat{\beta}_j' z_t$, i.e. $\hat{\beta}' S_{11} \hat{\beta} = I$, where S_{11} is the product moment matrix of z_{t-1} corrected for Δz_{t-1} and D_t.

(c.iii) The ordering given by the maximal conditional correlation with the stationary process Δz_t.

If we consider all three criteria relevant as identifying conditions then the estimated cointegrating relations as given in Table 1 are uniquely determined. Because the maximum likelihood estimator of the long–run relations is sometimes criticized for its lack of uniqueness it is of some interest to discuss whether the three conditions (or rather the last two, since stationarity is clearly mandatory) are reasonable from an economic point of view. And if the answer is negative what are the consequences.

The conditional independence condition (c.ii) is essentially the consequence of the chosen normalization, $\hat{\beta}' S_{11} \hat{\beta} = I$. Though it can be considered arbitrary in some sense (since we can also choose other normalizations) it arises as a natural consequence of the analysis of the likelihood function. Since empirical experience with a wide variety of economic applications shows that the unrestricted eigenvectors $\hat{\beta}_i$, i = 1,...r, surprisingly often can be given a direct interpretation in terms of the hypothetical relations, it is of some interest to ask whether this natural statistical choice can be justified in terms of a natural economic interpretation. We will give a somewhat heuristic answer here and leave a formal treatment for later research.

If the empirical problem is about macroeconomic behaviour in a market where equilibrating forces are allowed to work without binding restrictions, at least not in the long–run, one would generally expect at least two types of agents with disparate goals interacting in such a way that equilibrium is restored once it has been violated. These can be demanders versus suppliers, producers versus consumers, employers versus employees, etc. Therefore it seems plausible to assume that the long–run structure, $\beta' z_t$, should contain evidence about at least two fundamental behavioral relations. But the question remains whether it is empirically meaningful to assume that they are conditionally independent.

A somewhat heuristic guess is that the conditional independence often gives something interpretable when the information set contains i) variables that can formally and empirically identify the hypothetical long–run relations, ii) variables that can sufficiently well account for the short–run dynamic adjustment process. The first condition implies that for a demand and a supply relation, say, we have to include at least one variable which is (strongly) correlated with the demand and uncorrelated with the supply and vice versa. The second requirement implies that we have to condition on the short–run effects in order to statistically isolate the fundamental long–run effects. This in probably very important when the former effects are very different from the latter effects, which is likely to be the case when the adjustment to steady–state is hampered by costly information, political regulations, and other kinds of binding restrictions in the short–run.

Whether the third criterion (c.iii), the maximal correlation with the stationary part of the process, is economically relevant may, however, not be so obvious. Given stationarity and the conditional independence, it seems plausible that one might very well find a more satisfactory economic structure by rotation. Therefore, even if a direct interpretation of the unrestricted cointegration vectors is sometimes possible, the results should be considered indicative rather than conclusive, and cannot replace formal testing of structural hypotheses.

5.2. The estimates of the unrestricted cointegration space.

Considering the points discussed above we will now investigate the long–run structure given by the unrestricted β and α estimates presented in Table 1.

Table 1
The unrestricted estimates from the cointegration analysis of
$z_t = [m3, y, p, i_s, i_b, t]$, t = 1975:3 – 1991:1

The Eigenvalues $\hat{\lambda}_i$

.496	.353	.261	.177	.033

The Trace Test Statistics, $T\,\Sigma \ln(1-\hat{\lambda}_i)$

100.733	58.907	32.341	13.920	2.026

The stationary eigenvectors, $\hat{\beta}_i$, and the nonstationary eigenvectors, $\hat{v}_i$.

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{v}_4$	$\hat{v}_5$
m	–.186	.139	.026	1.000	1.000
y	1.000	–.235	.098	.783	.827
p	.212	.322	–.354	2.519	2.797
i_s	–.097	1.000	–.106	–1.320	5.837
i_b	.087	–1.432	1.000	–.684	–8.864
t	–.006	–.008	.005	–.085	–.109

The Weights

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$		
Δm	.105	.140	–.543	–.004	–.011
Δy	–.451	–.118	–.576	–.097	–.001
Δp	.316	–.047	–.013	–.071	–.002
Δi_s	.287	–.241	–.259	.045	.003
Δi_b	.129	.014	–.216	.011	.004

The Matrix $\hat{\Pi} = \hat{\alpha}\hat{\beta}$

	m	y	p	i_s	i_b	t
Δm	–.015	.019	.260	.188	–.735	–.005
	(.025)	(.118)	(.079)	(.097)	(.236)	(.001)
Δy	.053	–.479	.071	–.013	–.446	.001
	(.029)	(.134)	(.089)	(.109)	(.267)	(.002)
Δp	–.066	.326	.056	–.077	.082	–.002
	(.017)	(.079)	(.053)	(.065)	(.159)	(.001)
Δi_s	–.094	.318	.075	–.241	.111	–.001
	(.019)	(.088)	(.059)	(.072)	(.176)	(.001)
Δi_b	–.028	.105	.108	.024	–.225	–.002
	(.010)	(.048)	(.032)	(.039)	(.096)	(.001)

where the standard deviations of $\hat{\Pi}_{ij}$ are given in parentheses.

An inspection of the first three $\hat{\beta}$–vectors indicates that the first vector can approximately be given as:

$$y_t \approx .2(m_t - p_t) + .006t + \text{constant term}$$

i.e. essentially describes an aggregate demand relation for which real income grows around a linear trend with small positive effects of real money but hardly any direct interest rate effects. The lack of interest rate effects on y_t might seem odd but the interest rates influences y_t via the two remaining cointegration vectors. The second row of the Π–matrix, which gives the combined effects of all three cointegration vectors, shows that there are indeed negative interest rate effects on real income, in particular from the bond rate. This is an illustration of the case when a linear combination of the cointegration vectors corresponds to the hypothetical relation.

The second vector can approximately be described as:

$$i_s \approx 1.4 i_b - 0.322(p_t - 0.019t) + \text{constant term}^3$$

Note that since interest rates are measured as yearly rates, but price changes are quarterly, the coefficient to inflation rate is in fact $4 \times .322 = 1.3$. Thus, expressed in yearly rates:

$$i_s \approx 1.4 i_b - 1.3(p_t - 0.019t) + \text{constant term}$$

i.e. the short interest rate follows the long bond rate and the inflation rate, measured as the deviation of the price level from the average nominal growth given as a linear trend. The third vector is approximately given by:

$$i_b = 0.35 \times 4(p_t - 0.019t) + \text{constant term}$$

i.e. describing the nominal bond rate as a function of the inflation rate measured again as the price level around the linear trend.

The estimates of the corresponding α coefficients seem to support the suggested interpretation of the $\hat{\beta}$ vectors. Provided that the conditional independence can be considered relevant for the empirical problem, the above structure can be used as a starting point. The next step is then to test whether this structure can be simplified by imposing data consistent restrictions and to check whether some of these restrictions are identifying.

In case only the stationarity requirement is considered relevant, we have that the relations $\hat{\beta}_i' z_t$ are not uniquely determined, since for any choice of the relations we can find many more by taking linear combinations. Thus in terms of stationarity only the cointegrating space is well defined in the model. In this case an economist would like to discriminate between his relations using other criteria, such as zero–restrictions and homogeneity restrictions, albeit maintaining the stationarity restriction. Within the unrestricted β space it is possible to impose $s_i = r-1$ restrictions on each vector by taking linear combinations and consequently there would be no testing involved in this case. If the

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[3]The estimate of the average quarterly growth rate of prices is taken from Table 2.

number of restrictions $s_i > r-1$ we will have to test the corresponding reduction of the parameter space.

Whatever the situation, testing restrictions on the cointegration space is a crucial part of the analysis. We will discuss various cases below where the suggested restrictions do and do not imply testing and cases where some of the restrictions are identifying and some are not. In general we will use the rank test procedures to investigate whether the chosen information set is sufficiently large to allow us to discriminate between the hypothetical relations given by the ISLM model.

5.3. Exact identification.

It is useful for the discussion in this section to introduce the economic distinction of endogenous and exogenous variables, although maintaining the modeling of the full system. For instance let the vector z_t be partitioned into $z_t' = (y_t', x_t')$ where $y_t' = (m3, y, p)$ is the vector of supposedly endogenous variables and $x_t' = (i_s, i_b)$ is the vector of the supposedly exogenous variables. We choose the same number of endogenous variables as the number of cointegrating relations.

In the empirical illustrations below we will discuss different aspects of identification using the partitioning into y_t and x_t. We consider first two examples of exactly identified systems, corresponding to some very simple choices of β and α respectively.

Example 1. The long–run reduced form model:
In model (6) we decompose $\beta' = (\beta_1', \beta_2')$ with β_1 a square matrix. If β_1 has full rank, then we can normalize β as follows:

$$\alpha\beta' = \alpha\beta_1'\{1, \beta_1'^{-1}\beta_2\} = \tilde{\alpha}\tilde{\beta}'.$$

In this case the equations become:

$$\begin{bmatrix} \Delta y_t \\ \Delta x_t \end{bmatrix} = \Pi_1 \begin{bmatrix} \Delta y_{t-1} \\ \Delta x_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{\alpha}_1 \\ \cdots \\ \tilde{\alpha}_2 \end{bmatrix} \begin{bmatrix} I_m \vdots \tilde{\beta}_2' \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}. \tag{11}$$

such that the stationary relations are solved for the coefficients to y_t and becomes:

$$y_t + \tilde{\beta}_2' x_t = u_t.$$

If moreover $\tilde{\alpha}_2 = 0$, then the coefficient $\tilde{\beta}_2$ does not appear in the equation for Δx_t and x_t is weakly exogenous for $(\tilde{\alpha}_1, \tilde{\beta}_2)$. In this case efficient inference on the long–run relations can be conducted in the conditional model of Δy_t given Δx_t, see Johansen (1992a). Note that if $\tilde{\alpha}_2 = 0$, then necessarily the coefficient matrix of y_t will have full rank, which gives a possibility to test that the representation (11) can be used.

Table 2.
Exact Identification $\tilde{\beta}' = [I_m, \tilde{\beta}_2]$

$$\tilde{\beta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \quad H_1 = \begin{bmatrix} 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 0&0&0&0 \\ 1&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} \quad H_3 = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$$

The Eigenvectors $\tilde{\beta}_i$ — The Weights $\hat{\alpha}_i$

$$\begin{bmatrix} 1.000 & .000 & .000 \\ .000 & 1.000 & .000 \\ .000 & .000 & 1.000 \\ 6.341 & .871 & 1.011 \\ -3.076 & .151 & -3.009 \\ -.027 & -.007 & -.019 \end{bmatrix} \qquad \begin{bmatrix} -.015 & .019 & .260 \\ .053 & -.479 & .071 \\ -.066 & .326 & .056 \\ -.094 & .318 & .075 \\ -.028 & .105 & .108 \end{bmatrix}$$

The estimates of $\tilde{\beta}$ in example 1 are given in Table 2. and the estimates of β in example 2 are given in Table 3.

Table 3.
Exact Identification $\beta = \{H_1\varphi_1, H_2\varphi_2, H_3\varphi_3\}$

$$\beta = \begin{bmatrix} a & 0 & 0 \\ * & * & * \\ -a & * & * \\ 1 & 1 & 0 \\ 0 & -1 & 1 \\ * & * & * \end{bmatrix} \quad H_1 = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ -1&0&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \\ 0&0&0&1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&-1&0 \\ 0&0&0&1 \end{bmatrix} \quad H_3 = \begin{bmatrix} 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$$

The Eigenvectors $\hat{\beta}_i$ — The Weights $\hat{\alpha}_i$

$$\begin{array}{l} m3 \\ y \\ p \\ i_s \\ i_l \\ t \end{array} \begin{bmatrix} .174 & .000 & .000 \\ .077 & .720 & .364 \\ -.174 & .368 & -.314 \\ 1.000 & 1.000 & .000 \\ .000 & -1.000 & 1.000 \\ -.002 & -.011 & .003 \end{bmatrix} \qquad \begin{bmatrix} -.083 & .271 & -.464 \\ .300 & -.313 & -.759 \\ -.376 & .299 & .382 \\ -.536 & .295 & .406 \\ -.160 & .184 & -.041 \end{bmatrix}$$

Example 2. Other exactly identifying restrictions.
Here we have imposed the restrictions given by the design matrices H_1, H_2 and H_3 in Table 3. We have imposed exactly $s_i = r-1 = 2$ restrictions on each row, which implies that there is no testing involved. It is nevertheless relevant to ask whether the restrictions are identifying. This can be verified by checking the rank conditions given by (10) in section 4. The rank of the appropriate matrices are given in column a of Table 5. The conditions are satisfied and we conclude that the restrictions in Table 3 are just identifying.

5.4. Overidentification:

Here we consider restrictions that reduce the parameter space, the so called overidentifying restrictions. We will discuss two examples in which the design matrices H_1, H_2 and H_3 are constructed from economic considerations, where it turns out in the first example, that while the restrictions reduce the parameter space the identifying property is lost, but easily repaired. The second example satisfies the formal identification criterion (10).

Example 1. The hypothesis that we want to investigate is whether the data is consistent with a money demand relation, the interest rate differential and the deviation of the bond rate from a measure of the inflation rate. The H matrices describing the restrictions are given in Table 4 together with the estimated values of the restricted coefficients.

Formal identification requires that $\text{rank}(R_i'H_j) \geq 1$ for i,j = 1, 2, 3 and $j \neq i$, and that rank $(R_i'(H_j,H_m)) \geq 2$ for i,j,m different. The rank tests are given in table 5 column b where the i.j elements should be at least 1 and the i.jk elements at least 2 for formal identification. Thus the rank conditions are not satisfied simply because the space spanned by H_2 is contained in the space spanned by H_1. Thus there are at least two vectors that satisfy the restrictions of the first equation, and the first equation is thus not identified. The model specified by the restrictions in Table 4 is thus not identifying in the sense defined here.

This implies that the four parameters φ_{11}, φ_{12}, φ_{13} and φ_{14} cannot be estimated without further restrictions. Another way of expressing this is that one of the interest rates can be removed from β_1 by taking linear combinations with β_2. For instance, $\beta_1 + 1.89\times\beta_2$ removes the bond rate from β_1. In this set–up we can only estimate uniquely the impact of a linear combination of the interest rates in the first relation. Although the restrictions in Table 4 are not identifying, they are genuine restrictions on the parameter space and the model can be tested by a likelihood ratio test. The degrees of freedom in the test for restrictions is calculated as $\nu = \Sigma_i(p1-r+1-s_i)$ where p1 = 6, is given by the

Table 4
Lack of formal identification $\beta = \{H_1\psi_1, H_2\psi_2, H_3\psi_3\}$

$$\beta = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & * \\ * & 1 & 0 \\ * & -1 & 1 \\ * & 0 & * \end{bmatrix} \quad H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad H_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Test of overidentifying restrictions Q(5) = 22.55

The Eigenvectors $\hat{\beta}_i$

$$\begin{bmatrix} 1.000 & .000 & .000 \\ -1.000 & .000 & .000 \\ -1.000 & .000 & -.487 \\ 1.893 & 1.000 & .000 \\ 1.893 & -1.000 & 1.000 \\ -.005 & .000 & .009 \end{bmatrix}$$

The Weights $\hat{\alpha}_i$

$$\begin{bmatrix} -.021 & .205 & -.475 \\ -.013 & -.120 & -.224 \\ -.146 & .336 & .702 \\ -.072 & -.071 & .112 \\ -.022 & .097 & -.126 \end{bmatrix}$$

Table 5
Verification of the rank condition (10) of formal identification in the long–run structure.

$r_{i.j}$	a[1]	b[2]	c[3]	d[4]	$r_{i.jk}$	a	b	c	d
					Relation $H_1\varphi_1$				
1.2	2	0	1	1	*1.23*	2	2	2	3
1.3	2	1	2	2					
					Relation $H_2\varphi_2$				
2.1	2	3	3	3	*2.13*	2	4	4	5
2.3	1	3	3	2					
					Relation $H_3\varphi_3$				
3.1	2	2	2	2	*3.12*	2	1	2	3
3.2	1	1	1	1					

[1] Column a refers to the H_i restrictions of Table 3
[2] Column b refers to the H_i restrictions of Table 4
[3] Column c refers to the H_i restrictions of Table 4 where H_1 is replaced by (12)
[4] Column d refers to the H_i restrictions of Table 6

dimension $p1 \times r$ of β. Thus $\nu = (4-3) + (4-1) + (4-3) = 5$, since $p1-r+1 = 4$. Here s_i is the number of freely estimated parameters in β_i. Note that the ν becomes 5 and not 4, because there are only 3 freely estimated parameters in φ_1.

The test statistic for the restrictions in Table 4 becomes $Q(5) = 22.55$, which is clearly significant and the hypothetical structure $\beta = (H_1\varphi_1, H_2\varphi_2, H_3\varphi_3)$ given in Table 4 is rejected. This means that the data does not support the existence of a long–run money relation with both price and income homogeneity. If we instead put the bond rate coefficient to zero in β_1, the matrix H_1 is changed to

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{12}$$

and we get the ranks given in column c in Table 5. In this case we have formal identification of the new restrictions without loosing degrees of freedom, and of course the same test statistic as before.

Example 2. The hypothesis that we want to test here is whether the data is consistent with a domestic demand relation together with the interest rate differential and the real bond rate. The H matrices are given in Table 6, together with the estimates. The question of formal identification can again be checked by investigating the conditions of Theorem 1, and we find the results in column c Table 5. The condition for formal identification is thus satisfied. The test for overidentifying restrictions $Q(5) = 3.5$ clearly shows that the imposed restrictions describe the data well. The next question is then whether we have economic identification, that is, are the estimated coefficients economically meaningful?

The first relation is a proxy for aggregate demand around a linear trend (yearly growth rate is appr. 2.1%) with positive real money effects. The long–run real money effect might seem implausible. Nevertheless it can be given an economic interpretation in Keynesian economics, assuming that nominal wages do not rise as much as prices during a monetary expansion. This is however based on the assumption that prices rise in proportion to a monetary expansion, for which there is little empirical support in our data, as the subsequent analysis will show. The second relation tells us that the interest rate differential is stationary and the third relation describes the real long–term bond rate where the expected inflation rate is proxied by the deviation from the average linear growth of the prices. The final check whether the suggested long–run structure is economically meaningful has to be decided in terms of the estimated α coefficients, which generally should be discussed w.r.t a structural formulation of the short–run part of the model, i.e. in terms of (7).

Table 6.
Overidentification $\beta = \{H_1\varphi_1, H_2\varphi_2, H_3\varphi_3\}$

$$\beta = \begin{bmatrix} a & 0 & 0 \\ 1 & 0 & 0 \\ -a & 0 & * \\ 0 & 1 & 0 \\ 0 & -1 & 1 \\ * & 0 & * \end{bmatrix} \quad H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad H_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad H_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Test of overidentifying restrictions $Q(5) = 3{,}5 \sim \chi^2(5)$

The Eigenvectors $\hat{\beta}_i$

$$\begin{bmatrix} .193 & .000 & .000 \\ 1.000 & .000 & .000 \\ .193 & .000 & -.488 \\ .000 & 1.000 & .000 \\ .000 & -1.000 & 1.000 \\ -.005 & .000 & .009 \end{bmatrix}$$

The Weights $\hat{\alpha}_i$

$$\begin{bmatrix} .030 & \mathbf{.159} & \mathbf{-.569} \\ \mathbf{-.458} & -.001 & \mathbf{-.405} \\ \mathbf{.325} & -.039 & .054 \\ \mathbf{.337} & \mathbf{-.308} & \mathbf{-.168} \\ \mathbf{.109} & .023 & \mathbf{-.213} \end{bmatrix}$$

In Table 6 the short–run reduced form α's give some indication of the appropriateness of the structural specification. To increase readability we have marked the interesting α_{ij} coefficients with bold face. They indicate that money stock reacts positively to changes in the interest rate differential, α_{12} (the own yield effect) and negatively to changes in real bond rate, α_{13} (the opportunity cost). For real aggregate demand there is a strong error–correction effect, α_{21} and a strong negative effect of real bond rate, α_{23}. The inflation rate is positively related to excess aggregate demand, α_{31} and the three months commercial rate reacts positively to excess aggregate demand, α_{41} (the government reaction to an overheated business cycle) and follows the nominal bond rate, α_{42}, as well as the real bond rate, α_{43}. Finally the nominal bond rate is adjusting to a stationary real rate, α_{53}, possible with some minor effects from excess aggregate demand, α_{51}.

6. Identification of the short–run model structure.

In many applications prior economic hypotheses are less precise about the short–run structure than about the long–run structure. Much less is usually known in advance about the adjustment mechanisms. This means that in many cases the identification of the short–run structure has more the character of data analysis aiming at the "identification" of a parsimonious parameterization than testing well–specified economic hypotheses. This does not mean that the short–run analysis is unimportant in any sense. It is often the case that the econometric analysis of the short–run structure leads to new hypotheses about the mechanisms of the short–run adjustment behaviour, which subsequently can lead to more precise starting hypotheses when analyzing similar problems based on other data sets.

For this empirical application we have no strong prior hypotheses about the short–run structure and the identification process will to a certain extent be explorative. In addition to formal and empirical identification, we will require plausible estimates of the short–run structure before it can qualify as a candidate for the "final" econometric model. This is stated as plausible signs of derivatives, short–run homogeneity restrictions, but, as will be demonstrated below, above all as plausible estimates of the adjustment coefficients, α, to the identified long–run relations. Since the adjustment coefficients associate the short–run structure with the long–run structure they are essentially the cornerstones in the identification process.

6.1. Different parameterizations of the set λ_s.

We will begin the analysis of the short–run structure by investigating three different parameterizations of the statistical model as defined by (6). As in section 5.1 we will discuss under which conditions the parameters are well–defined. In all subsequent discussions the long–run relations, $\hat{\beta}_j' z_{t-1}$, are fixed by the estimates given in Table 6.

6.1.1. The unrestricted reduced form (URF) and the conditional expectations form (CEF)

The estimates of the URF model are given in Table 7. The coefficients of each row are given by the conditional expectation $E(\Delta z_{it} \mid \Delta Z_{t-1}, \beta' Z_{t-1}, D_t)$, i = 1,...,5. The current effects between the variables of the system are given by the correlation matrix R. To increase readability coefficients with an absolute t–value > 1 have been indicated by bold. The following can be noted: i) the short–run structure is as expected overparameterized with many insignificant coefficients, ii) the adjustment coefficients α_{ij} seem to provide the bulk of explanatory power, iii) the signs and the magnitudes of the α_{ij} coefficients seem reasonable, whereas the adjustment coefficients to the lagged changes of the process are more difficult to interpret, iv) some of the correlations in R are quite large suggesting that we need to specify the structure of simultaneous effects in A_0. This structure will be analyzed below.

The estimates of the conditional expectations $E(\Delta z_{it} \mid \Delta z_{jt}, j \neq i, \Delta Z_{t-1}, \beta' Z_{t-1}, D_t)$, i = 1,...,5 are given in Table 8. These are estimated by linear regression equation by equation. Although not necessarily the most interesting representation the parameters

TABLE 7

Unrestricted estimates of the reduced form

	I					Γ_1					α		
	Δi_s	Δy_t	Δm_t	Δp_t	Δi_b	Δi_{st-1}	Δy_{t-1}	Δm_{t-1}	Δp_{t-1}	Δi_{bt-1}	$\hat\beta_1' z_{t-1}$	$\hat\beta_2' z_{t-1}$	$\hat\beta_3' z_{t-1}$
Δi_s	-1					.33 (2.2)	-.06 (0.6)	**.29** (3.2)	**-.23** (2.1)	**-.49** (1.5)	**.34** (3.5)	**-.29** (3.8)	-.10 (0.8)
Δy_t		-1				-.13 (0.8)	-.02 (0.1)	.06 (0.4)	.13 (0.5)	.34 (0.7)	**-.42** (2.7)	-.09 (0.7)	**-.31** (1.5)
Δm_t			-1			.06 (0.3)	-.01 (0.1)	**.28** (2.2)	**.41** (2.7)	.08 (0.2)	-.03 (0.2)	**.16** (1.5)	**-.44** (2.4)
Δp_t				-1		.00 (0.0)	**-.19** (1.6)	.04 (0.3)	**-.17** (1.3)	-.08 (0.2)	**.30** (2.6)	**-.15** (1.7)	**.46** (3.0)
Δi_b					-1	-.01 (0.1)	-.04 (0.7)	**.10** (1.9)	-.01 (0.1)	.00 (0.0)	**.14** (2.6)	**-.05** (1.1)	**-.10** (1.3)

$\hat\beta_1' z_{t-1} = y - 0.19(m-p) - .00526t - 0.027D84 - 8.43$

$\hat\beta_2' z_{t-1} = i_s - i_b + .00967D84 + 0.03$

$\hat\beta_3' z_{t-1} = i_b - .488(p-0.019t) - 0.008D84 - 0.52$

$$R = \begin{bmatrix} 1 & -.13 & .61 & -.06 & .02 \\ & 1 & .04 & .29 & .30 \\ & & 1 & -.06 & .06 \\ & & & 1 & .34 \\ & & & & 1 \end{bmatrix}$$

TABLE 8

Unrestricted estimates of the full model

	A_0					A_1					a		
	Δi_s	Δy_t	Δm_t	Δp_t	Δi_b	Δi_{st-1}	Δy_{t-1}	Δm_{t-1}	Δp_{t-1}	Δi_{bt-1}	$\hat\beta_1' z_{t-1}$	$\hat\beta_2' z_{t-1}$	$\hat\beta_3' z_{t-1}$
Δi_s	-1	**-.11** (1.4)	.02 (.2)	.02 (.2)	**1.08** (5.4)	**.36** (2.9)	-.01 (.2)	**.19** (2.4)	**-.21** (2.2)	**-.46** (1.7)	**.14** (1.4)	**-.24** (3.7)	-.03 (.2)
Δy_t	**-.38** (1.4)	-1	**.23** (1.4)	**.31** (1.6)	**.53** (1.1)	**.26** (1.1)	.04 (.3)	.06 (.4)	.00 (.0)	.15 (.3)	**-.44** (2.6)	**-.16** (1.2)	**-.33** (1.3)
Δm_t	.05 (.2)	**.18** (1.4)	-1	**.34** (2.0)	-.28 (.6)	.02 (.1)	.05 (.3)	**.29** (2.2)	**.45** (2.8)	.09 (.2)	-.03 (.2)	**.23** (1.9)	**-.56** (2.7)
Δp_t	.05 (.2)	**.17** (1.6)	**.24** (2.0)	-1	.10 (.3)	-.06 (.3)	**-.18** (1.6)	**-.13** (1.2)	**-.28** (2.0)	-.14 (.4)	**.34** (2.8)	**-.16** (1.6)	**.63** (4.0)
Δi_b	**.36** (5.4)	**.05** (1.1)	-.03 (.6)	.02 (.3)	-1	**-.13** (1.8)	-.02 (.3)	.00 (0.0)	**.08** (1.5)	**.16** (1.1)	.03 (.6)	**.07** (1.6)	-.07 (.9)

$\hat\beta_1' z_{t-1} = y - 0.19(m-p) - .00526t - 0.027D84 - 8.43$

$\hat\beta_2' z_{t-1} = i_s - i_b + .00967D84 + 0.03$

$\hat\beta_3' z_{t-1} = i_b - .488(p-0.019t) - 0.008D84 - 0.52$

$$R = \begin{bmatrix} 1 & .20 & -.62 & -.03 & -.03 \\ & 1 & -.16 & -.21 & -.22 \\ & & 1 & .10 & -.04 \\ & & & 1 & -.29 \\ & & & & 1 \end{bmatrix}$$

$\{A_0, A_1, a, R, \hat{\sigma}_\epsilon\}$ are uniquely defined by the parameters θ. We note the following: i) the number of coefficients with an absolute t–value > 1 has increased substantially, but there are only a few strongly significant coefficients making the structure look rather diffuse, ii) the adjustment coefficients to the current and lagged changes of the process are easier to interpret, iii) there seem to be short–run simultaneity between the short– and long–term interest rate and between money and prices, iv) the residual correlations are essentially unchanged compared to the reduced form. These observations will be used to specify the third representation of the process, namely the unrestricted triangular form as discussed below.

6.1.3. The unrestricted triangular form (UTF)

The ordering of the variables in the triangular form can be chosen in p! different ways which in our case amounts to 120 possible representations. This of course is a good illustration of the arbitrariness inherent in this form unless prior information can be used to restrict the number of interesting representations. Here we have chosen the ordering given in Table 9 motivated by the following considerations:

Based on the CEF estimates in Table 8 there seems to be simultaneous correlation between Δi_s and Δi_b as well as between Δm_t and Δp_t. In a causal chain representation we have to make a decision about which variable is more likely to have currently caused the other, i.e. is Δi_{st} causing Δi_{bt} or vice versa and is Δm_t causing Δp_t or vice versa. Although this choice is always subjective in some sense, one can combine the statistical information given by the estimates in Table 7 and Table 8 with one's economic intuition to pick up a few candidates of the 120 possible ones that seem to be able to describe an interesting and plausible economic structure. A priori one would consider the long–term bond rate to mirror the state of the fundamentals of the domestic economy relative to the foreign economies i.e. to be determined relative to the foreign interest rates. If this is correct i_b would essentially be exogenously determined in our system and the causality go from i_b to i_s. This interpretation is also supported by the fact that in the URF and the CEF models the adjustment coefficient to the interest rate differential, $\beta_2' z_{t-1}$, is large and negative in the equation for Δi_s, whereas it is insignificant in the equation for Δi_b. The estimated coefficients measuring short–run adjustment to the changes of the process as given in Table 8 seem to support this interpretation too; the short–term interest rate seems to adjust strongly to changes in the long–term bond rate with a certain autoregressive pattern, and to changes in the real money stock, whereas the estimated coefficients in the long bond rate equation are much smaller and more diffuse. Therefore we choose Δi_{st} to be caused by Δi_{bt} in the triangular form representation below.

Now to the choice of ordering of Δm_t and Δp_t. According to standard macroeconomic theory, an increase in money stock will cause prices to rise and consequently the causality should go from money to prices. The empirical verification of this rule has however not been very successful and it is still highly debatable whether money matters or not, and in case money matters it is still more debatable whether prices react quickly or slowly. Keynesian macroeconomics usually assume sticky prices, whereas neoclassical macroeconomics assume flexible prices. This is a typical example where economic theory cannot give an unambiguous guidance for the choice of causal ordering and one has to rely more on the statistical information in the data. From Table 8 we notice that Δm_t reacts to current and lagged changes in the price such that the assumption of short–run price homogeneity is almost exactly fulfilled, whereas the total impact on Δp_t from current and lagged changes in nominal money is very small. This seems to give strong empirical support for the choice of Δp_t causing Δm_t and not the other way around.

Finally what remains is the choice of ordering between Δi_{st}, Δy_t and Δp_t. In the Keynesian version of the closed economy ISLM model, the interest rate and the income would be chosen as endogenous variables, using the assumption of sticky prices when the economy is not on the full employment income level. In the corresponding neoclassical model version, the interest rate and the price would be chosen as the endogenous variables, based on the assumption of flexible prices. In both models money supply would be assumed to be exogenously given. Since empirically the demand for money has usually been shown to be endogenously determined, i.e. it adjusts to a steady–state level which is given by the other determinants of the system, it becomes now relevant to discuss whether money stock as measured by m3 can be considered supply determined or demand determined. As appears from the discussion in Juselius (1992a), m3 can be considered to be to some degree controllable by the monetary authorities and therefore one can argue that the observed values of m3 is the outcome of supply restrictions rather than demand considerations. It is however debatable whether in practice the Federal Reserve Bank of Australia has been able to exert control over money stock, in particular in the light of the capital deliberation in the eighties, (see Milbourne (1990) for a more extended discussion). Whether the observed value of m3 is demand or supply determined is therefore essentially an empirical question to which it is not easy to get an unambiguous answer. From Table 7 and Table 8 it can be seen that the estimated adjustment coefficient to $\beta_1'z_{t-1}$ in the money equation is essentially zero, implying that money stock measured by m3 does not adjust directly to a disequilibrium in this relation, whereas it does seem to adjust to disequilibrium in the interest rate differential measured by $\beta_2'z_{t-1}$, and deviations from the real bond rate as measured by $\beta_3'z_{t-1}$. This seems broadly to support the exogeneity assumption of the ISLM model. Thus we conclude that this system primarily determines Δi_s and Δy_t, followed by Δm_t, Δp_t and Δi_b. Whether we choose Δi_{st} or Δy_t at the top of the triangular system does not seem to be much of a difference.

TABLE 9

Unrestricted estimates of the triangular form

	A_0					A_1					a		
	Δi_s	Δy_t	Δm_t	Δp_t	Δi_b	Δi_{st-1}	Δy_{t-1}	Δm_{t-1}	Δp_{t-1}	Δi_{bt-1}	$\hat{\beta}_1'z_{t-1}$	$\hat{\beta}_2'z_{t-1}$	$\hat{\beta}_3'z_{t-1}$
Δi_s	-1	**-.11** (1.4)	.02 (.2)	.02 (.2)	**1.08** (5.4)	**.36** (2.9)	-.01 (.2)	**.19** (2.3)	**-.21** (2.2)	**-.45** (1.7)	**.13** (1.4)	**-.24** (3.7)	-.03 (.2)
Δy_t		-1	**.24** (1.5)	**.31** (1.6)	.12 (.4)	.13 (.6)	.05 (.3)	-.01 (0.0)	.09 (.5)	.34 (.7)	**-.52** (3.1)	-.07 (.5)	**-.33** (1.3)
Δm_t			-1	**.42** (2.6)	-.22 (.6)	.01 (.0)	.06 (.4)	**.31** (2.5)	**.48** (3.2)	.13 (.3)	-.13 (0.9)	**.21** (2.0)	**-.65** (3.3)
Δp_t				-1	.14 (.5)	.01 (.0)	**-.19** (1.6)	.05 (.4)	**-.17** (1.3)	-.08 (.2)	**.28** (2.3)	**-.15** (1.7)	**.46** (3.0)
Δi_b					-1	-.01 (.1)	-.04 (.7)	**.10** (1.9)	.01 (.1)	.00 (.0)	**.14** (2.1)	**-.05** (1.1)	**-.10** (1.3)

$\hat{\beta}_1'z_{t-1} = y - 0.19(m-p) - .00526t - 0.027D84 - 8.43$

$\hat{\beta}_2'z_{t-1} = i_s - i_b + .00967D84 + 0.03$

$\hat{\beta}_3'z_{t-1} = i_b - .488(p-0.019t) - 0.008D84 - 0.52$

TABLE 10

Lack of empirical identification

	A_0					A_1					α		
	Δi_s	Δy_t	Δm_t	Δp_t	Δi_b	Δi_{st-1}	Δy_{t-1}	Δm_{t-1}	Δp_{t-1}	Δi_{bt-1}	$\hat{\beta}_1'z_{t-1}$	$\hat{\beta}_2'z_{t-1}$	$\hat{\beta}_3'z_{t-1}$
Δi_s	-1	.52 (0.0)	2.39 (0.1)	8.40 (0.1)	-32.7 (0.1)	**0**	**0**	3.59 (0.1)	**0**	**0**	2.90 (0.1)	-1.18 (0.1)	-6.65 (0.1)
Δy_t	**0**	-1	3.80 (0.2)	**0**	**0**	-.09 (0.1)	.03 (0.0)	-1.0 (0.2)	-1.43 (0.2)	**0**	-.29 (0.5)	-.68 (0.3)	1.36 (0.2)
Δm_t	**0**	**0**	-1	.06 (0.1)	**0**	.06 (0.4)	**0**	**.28** (2.3)	**.42** (2.7)	.08 (0.2)	-.05 (0.3)	**.16** (1.4)	**-.47** (1.3)
Δp_t	.07 (0.1)	.06 (0.1)	-.36 (0.6)	-1	**0**	**0**	**-.19** (1.3)	.04 (0.1)	**0**	**0**	.23 (0.6)	-.08 (0.6)	**.29** (1.0)
Δi_b	.01 (0.0)		**0**	**0**	-1	-.02 (0.1)	-.05 (0.8)	.10 (0.9)	-.01 (0.0)	**0**	**.15** (1.2)	-.05 (0.4)	**-.11** (1.1)

$\hat{\beta}_1'z_{t-1} = y - 0.19(m-p) - .00526t - 0.027D84 - 8.43$

$\hat{\beta}_2'z_{t-1} = i_s - i_b + .00967D84 + 0.03$

$\hat{\beta}_3'z_{t-1} = i_b - .488(p-0.019t) - 0.008D84 - 0.52$

6.2. The simultaneous system of equations:

It appears from the estimates of the standardized covariances $\sigma_{ij}(\sigma_{ii}\sigma_{jj})^{-\frac{1}{2}}$ given in Table 7 that there are simultaneous effects between changes in Δi_{st} and Δi_{bt}, and between Δm_t and Δp_t, and to some extent also between Δy_t, Δp_t and Δm_t. In the triangular form above the system was identified by imposing zero restrictions on the off–diagonal elements of the residual covariance matrix and zero restrictions on the elements below the diagonal of A_0. The latter restrictions were motivated by economic arguments, but the assumption of uncorrelated residuals seems more ad hoc and is primarily motivated by computational convenience. In this section we will now relax these restrictions and estimate the model as a system of five simultaneous equations. We will first impose p–1 = 4 restrictions on each equation and investigate whether the system is formally identified. The system will then be estimated in the exactly identified form and empirical identification is investigated. Using these results overidentifying restrictions are imposed on the system and formal identification investigated in the new model. The imposed restrictions are tested against the data and empirical and economic identification are discussed.

6.2.1. Exactly identifying restrictions.

The p–1 = 4 zero restrictions imposed on each equation appears from Table 11 and the tests for formal identification of this system are given in Table 10.

It appears that the conditions for identification as given by (10) are satisfied and estimation can proceed. The estimated coefficients are given in Table 11 with the calculated t–values in parentheses. These are for most coefficients extremely small, indicating lack of empirical identification.

Based on the URF and CEF estimates it seems likely that the lack of empirical identification is due to the fact that with the chosen information set we cannot discriminate between the bond rate and the three months interest rate. From Table 7 it appears that the bond rate hardly reacts to the lagged changes of the process, whereas Δi_s clearly does. From Table 8 it appears that in the conditional expectations form of Δi_{bt} the regression estimates are essentially proportional to corresponding estimates in the equation of Δi_s. As discussed in section 5 and formalized by Definition 2, an economic model is empirically identified if its parameter λ is not contained in any nonidentified model. This will now be investigated by imposing such overidentifying restrictions on the equation Δi_b that are consistent with the reduced form estimates. Thus the equation for Δi_b is given by:

$$\Delta i_{bt} = a_1 \Delta m_{t-1} + a_2 \hat{\beta}_1{}' z_{t-1} + a_3 \hat{\beta}_3{}' z_{t-1} + \psi D_t + \text{error} \tag{13}$$

whereas the other equations are given as in Table 10. In Table 11 column b the rank condition for formal identification of the system is checked. It can now be seen that the

Table 11.
Verification of the rank condition (10) for formal identification in the short–run structure.

$r_{i.j}$	a[1]	b[2]	c[3]	$r_{i.jk}$	a	b	c	$r_{i.jkl}$	a	b	c	$r_{i.jklm}$	a	b	c
							Equation 1, Δi_{st}.								
1.2	3	3	3	1.23	3	3	3	1.234	4	4	4	1.2345	4	4	4
1.3	3	0	3	1.24	4	4	4	1.235	3	3	3				
1.4	4	4	3	1.25	3	3	3	1.345	4	4	4				
1.5	1	1	1	1.34	4	4	4								
				1.35	3	1	3								
				1.45	4	4	3								
							Equation 2, Δy_t.								
2.1	3	3	3	2.13	3	3	3	2.134	3	3	4	2.1345	4	4	4
2.3	3	1	2	2.14	4	4	4	2.135	3	4	3				
2.4	4	1	2	2.15	3	3	3	2.345	4	4	4				
2.5	1	2	1	2.34	3	2	4								
				2.35	3	3	3								
				2.45	3	3	2								
							Equation 3, Δi_{bt}.								
3.1	3	5		3.12	3	8	3	3.124	4	9	4	3.1245	4	9	4
3.2	2	5		3.14	4	9	4	3.125	3	8	3				
3.4	2	6		3.15	3	6	3	3.245	4	9	4				
3.5	3	6		3.24	4	7	4								
				3.25	3	8	3								
				3.45	4	9	3								
							Equation 4, Δm_t.								
4.1	4	5	3	4.12	4	8	4	4.123	4	4	4	4.1235	4	4	4
4.2	1	5	2	4.13	4	9	4	4.125	4	4	4				
4.3	2	6	3	4.15	4	6	3	4.245	4	4	4				
4.5	3	6	0	4.23	3	7	4								
				4.25	3	8	2								
				4.35	4	9	3								
							Equation 5, Δp_t.								
5.1	1	1	4	5.12	3	3	6	5.123	3	3	6	5.1234	4	4	7
5.2	2	2	4	5.13	3	1	6	5.124	4	4	7				
5.3	3	1	4	5.14	4	4	6	5.234	4	4	7				
5.4	2	3	3	5.23	3	3	6								
				5.24	3	3	5								
				5.34	4	4	6								

[1] column a corresponds to the system of equations in Table 10
[2] column b corresponds to the system of equations in Table 10 where eq Δi_b is given by (13)
[3] column c corresponds to the system of equations in Table 10 where eq. Δp is given by (14)

conditions for formal identification is not satisfied in this submodel for which the condition is violated w.r.t. eq. Δi_s relative to Δi_b, and w.r.t. Δi_s relative to Δi_b and Δp_t. This is an indication that with our chosen information set it is not possible to discriminate between the two interest rates. What seems to be needed is a variable that is strongly correlated with the bond rate without affecting the short interest rate. An possible candidate for this could be foreign interest rates and exchange rates. Such an analysis is, however, outside the scope of this paper. Since it seems difficult to achieve identification by imposing restrictions on the coefficients, one has the possibility of imposing restrictions on the covariance matrix. In this case identification can be achieved by assuming zero correlation between the residuals from the bond rate equation and the residuals from the short rate equation. The estimation of the system can now be proceeded by treating the four remaining equations as jointly determined, while the equation for the bond rate is determined by the reduced form estimates in Table 7.

Various attempts to estimate the four remaining equations simultaneously were unsuccessful in terms of large standard errors of estimates, which indicated that there was still lack of empirical identification. The reason for this seemed to be related to the second pair of variables with simultaneous effects, namely Δm_t and Δp_t. To investigate this we performed a similar analysis as for the interest rates. We kept the system of equations as given by Table 10, but imposed zero–restrictions on the equation for Δp_t, consistent with the reduced form estimates:

$$\Delta p_t = a_1\hat{\beta}_1'z_{t-1} + a_2\hat{\beta}_2'z_{t-1}\ a_3\hat{\beta}_3'z_{t-1} + \psi D_t + \text{error} \tag{14}$$

The test results are given in Table 11 column c. and it is now easy to verify that the conditions for formal identification is violated for eq. Δm_t w.r.t. Δp_t. Based on this result the estimation of the system should proceed by treating Δi_{st}, Δy_t and Δm_t as jointly determined, whereas the short–run determination of Δi_{bt} and Δp_t can be considered to take place outside this system.

6.2.2. An empirically identified system.

Based on the results of the previous analyses the system is estimated by treating Δi_{st}, Δy_t and Δm_t as jointly determined, whereas Δi_{bt} and Δp_t are considered to be determined outside this system. The imposed restrictions appear from Table 13 and the tests for formal identification for the jointly estimated equations are presented in Table 12. Since it appears that the three–dimensional system is formally identified it can now be estimated. This was done with the FIML procedure in PC–GIVE, see Hendry (1989).

The majority of the coefficients are now clearly significant and there does not seem to be any indication of lack of empirical identification. The likelihood ratio test of 13 overidentifying restrictions was 4.82 which is a strong support for the imposed structure.

Table 12.
Verification of the rank condition for formal identification of the short–run structure of Table (13)

	$r_{i.j}$		$r_{i.jk}$	
Equation 1, Δi_{st}:	*1.2*	4	*1.23*	5
	1.3	4		
Equation 2, Δy_t:	*2.1*	4	*2.13*	5
	2.3	3		
Equation 3, Δm_t:	*3.1*	4	*3.12*	5
	3.2	3		

TABLE 13

Equation system in over-identified form
LR-test for overidentifying restrictions Q(13) = 4.82

	A_0					A_1					a		
	Δi_s	Δy_t	Δm_t	Δp_t	Δi_b	Δi_{st-1}	Δy_{t-1}	Δm_{t-1}	Δp_{t-1}	Δi_{bt-1}	$\hat{\beta}_1'z_{t-1}$	$\hat{\beta}_2'z_{t-1}$	$\hat{\beta}_3'z_{t-1}$
Δi_s	-1	0	0	0	1.10 (6.5)	.34 (3.2)	0	.21 (3.0)	-.24 (2.8)	-.45 (2.1)	.19 (2.8)	-.28 (4.7)	0
Δy_t	0	- 1	.25 (1.1)	.31 (2.0)	0	.17 (1.3)	0	0	0	0	-.44 (4.1)	0	-.28 (1.3)
Δm_t	0	0	- 1	.35 (2.6)	0	0	0	.31 (2.9)	.41 (3.4)	0	0	.20 (2.3)	-.55 (3.6)
Δp_t	0	0	0	- 1	0	0	0	-.08 (0.8)	-.13 (1.0)	0	.20 (2.2)	-.12 (1.4)	.48 (3.4)
Δi_b	0	0	0	0	- 1	0	0	.08 (1.7)	0	0	.12 (3.0)	0	-.09 (1.3)

$\hat{\beta}_1'z_{t-1}$ = y - 0.19(m-p) - .00526t - 0.027D84 - 8.43

$\hat{\beta}_2'z_{t-1}$ = i_s - i_b + .00967D84 + 0.03

$\hat{\beta}_3'z_{t-1}$ = i_b - .488(p-0.019t) - 0.008D84 - 0.52

The stability of the model has been investigated by the recursive procedure in the FIML package of PCGIVE. Except for one outlier in 1990:1 in the m3 equation there were no obvious signs of lack of stability. Therefore the estimated system of Table 13 will be considered to be a sufficiently good approximation of the underlying data generating process.

It is interesting to notice that the short–run adjustment to the long–run relations provide us with the bulk of information that helps to discriminate between the equations. This is a strong indication that the identified long–run relations provide an economically meaningful structure of the DGP. Without using the information given by the cointegration properties of the data, only a small part of the variation given by the changes of the process would have been explained and the empirical analysis would hardly have provided much interesting results.

We will proceed to the last step in the identification process, i.e. the investigation of economic identification. Can the estimates be considered economically plausible?

The dominating feature of the three month interest rate equation is the strong dependence on the 10 year bond rate. The short–run interest rate seems to adjust almost proportionally not just to the level of bond rate, but also to the short–run changes in the bond rate. This can be seen from the estimated short–run dynamic impact:

$$a_b = (1.10 - .45) \;/\; (1 - .34) \approx 1$$

It can also be noticed that Δm_{t-1} and Δp_{t-1} have approximately equal coefficients with opposite signs, indicating that increases in real money stock tend to increase short interest rates. Finally excess aggregate demand tends to increase the short interest rate, which is consistent with the predictions from the ISLM model.

In the income equation only the error–correction mechanism as given by $\hat{\beta}_1{}'z_{t-1}$ is strongly significant. There is some indication that a monetary expansion has a direct positive effect on real income, although it is not very precisely estimated. The effect from real bond rate is negative as expected but not very significant. Altogether the information set used in this analysis does not seem sufficiently complete to estimate a good structural model for aggregate income. In particular a variable measuring the international competitiveness would be needed to account for the open economy impact on aggregate income determination.

In the money stock equation we can note that nominal money seems to have grown somewhat more than proportionally to current and lagged price changes:

$$a_m = (.35 + .41) \;/\; (1 - .35) \approx 1.1$$

It also reacts positively to the interest rate differential between short–rate and long–rate, where three months commercial rate is closely related to the average deposit rate on time deposits in the private banks and therefore can be considered a proxy for own interest rate.

As expected the real bond rate, as a measure of the opportunity cost of holding money has a strong negative effect on money stock (the portfolio effect).

In the bond rate equation we can note the expected positive effect from excess aggregate demand, and a rather small error–correction effect from deviations from the real bond rate.

In the price equation one can find a strong support for the positive impact of excess aggregate demand on the inflation rate. A positive interest rate differential seems to have a depressing effect on inflation rate, whereas nominal bond rates in excess of price inflation seem to have a strong positive impact on the inflation rate.

Altogether it seems reasonable to conclude that the estimates are plausible from an economic point of view and therefore that the estimated system can be considered a satisfactory approximation to the underlying economic structure, both in the short–run and the long–run. The policy implications of this empirical analysis seems to point to the ineffectiveness of monetary policy. None of the two possible instruments, money stock and short–term interest rate, seems to be ideal for a strong monetary regime. The impact of changes in m3 on prices seems very small indeed, and in general the impact of m3 on the determinants of this system is very modest. The short–term interest rate relative to the long seems to have a negative impact on inflation rate, but since the adjustment to the long interest rate seems to be quite fast there is not much room for using the short–term interest rate as an efficient policy instrument.

6. Discussion.

An economist usually makes a distinction between the <u>endogenous</u> variables, y, which are the variables of primary interest, and the <u>exogenous</u> variables, x, which are assumed to be be the main determinants of y. This classification is primarily motivated by economic arguments and often stems from a static comparative analysis in which the endogenous variables are solved for the exogenous which are assumed given.

For a statistician on the other hand it is natural to distinguish between the <u>stochastic</u> and the <u>nonstochastic</u> variables. Some of the stochastic variables may correspond to the endogenous variables of the economist, others to the exogenous variables. The economist would usually be prepared only to build a stochastic model for the endogenous variables, whereas the statistician would consider the model to be only partially specified without a full statistical model for all the stochastic variables inclusive the assumed exogenous variables.

Another important statistical classification is between <u>stationary</u> versus <u>nonstationary</u> variables. Among the latter one can distinguish between first order nonstationary variables, second order nonstationary variables, etc. which has the advantage that the data can be classified into homogeneous groups w.r.t. the variability of the trending behaviour.

The difference between the economists'and the statisticians' way of thinking can also be recognized in two fundamentally different approaches to empirical modeling: (i) the specific to general and (ii) the general to specific. The proponents of the first principle, usually preferred by people with a strong economic background, would start from a structural model based on precise economic hypotheses, derive the corresponding reduced form and ask questions about identification. This approach is based on the axiom of correct specification as was pointed out by Hendry and Richard (1983), because all statistical inference is invalid if the stochastic assumptions of the model are incorrect. In reality the economists are seldom omnipotent and the structural model is usually respecified in the light of empirical evidence until approximate fit is achieved. The proponents of the second principle, usually preferred by people with a strong statistical background, begin with a statistically well–specified model, often in the reduced form and then impose data consistent restrictions given by prior economic hypotheses.

The econometric approach of this paper is essentially influenced by the statisticians way of analyzing data, i.e. by starting with a general well–defined statistical model and then testing downwards (c.f. Hendry and Mizon (1990). It is motivated by the belief that in empirical model analysis the statistical concepts are less ambiguous than the economic concepts, since the former are directly related to the properties of the actual data. See for instance Summers (1991) for a discussion of the scientific illusion in empirical macroeconomics.

In this paper we have shown that the statistical classification of the process into stationary and nonstationary components can be utilized for the economic classification into long–run relations and short–run adjustment. Within this framework we have discussed the question of identification viewed from the two angles, namely the identification of the long–run structure versus the identification of the short–run structure. The concept of identification has traditionally been discussed under the axiom of correct specification. In such a world the structure of the model is known and identification is only the issue of verifying that the parameters of the model can be uniquely estimated. When one replaces the axiom of correct specification with the assumption of a reasonably well–structured DGP, with an underlying economic structure that can be consistent with several theories, it becomes important to widen the concept of identification. We have proposed that one should distinguish between i) formal identification, that relates to a hypothetical model, ii) empirical identification, that relates to the estimated parameter values and iii) economic identification, which relates to the economic interpretability of the estimated parameter values.

The concepts and the econometric approach have been illustrated by an application to Australian monetary behaviour. Though the aim of the initial analysis was to estimate a long–run money relation based on the conventional LM curve, the empirical analysis of the statistical model suggested that the ISLM model was a more appropriate framework for the underlying economic structure. The number of economic questions (hypotheses) was consequently enlarged relative to what was initially considered relevant. The hypotheses about the long–run structure were motivated by the Keynesian and the neoclassical versions of the static ISLM model, whereas the hypotheses about the short–run structure were motivated by various versions of buffer stock theories. A short summary of the basic findings are given below:

In the long–run structure we found three relations; the first measuring aggregate demand around a linear growth trend with some positive real money effects, the second measuring short–term interest rate as a function of long–term bond rate and the third measuring the real long–term bond rate. In the short–run structure we found that with the chosen information set we could only identify three simultaneous structural equations, i.e. equations for the short–term interest rate, real income and money stock, whereas price inflation and long–term bond rate were estimated in reduced form.

Generally the empirical analysis seems to point to the ineffectiveness of monetary policy in Australia at least by controlling the two instruments, money stock and interest rate. Even if the broad measure of money stock, M3, seemed to be largely exogenously for the long–run structure, the impact of changes in M3 on prices or the other determinants of the system was very modest. The differential between the short–term interest rate and the long–term bond rate seemed to have a negative impact on inflation, but since the adjustment to the long rate was relatively fast there did not seem to be much room for using the short rate as an efficient policy instrument. This conclusion is strengthened by the empirical support for the hypothesis that long–term bond rate is determined outside the domestic money market and therefore outside the control of the monetary authorities.

Finally the estimated results from the short–run analysis indicate that inflation rate is not much influenced by changes in money stock, but, however, is positively correlated with excess aggregate demand and with bond rate in excess of a stable long–term real rate. This seems to point at the Keynesian version of the ISLM model as a more satisfactory description of the aggregate behaviour for this sample period.

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