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TESTING WEAK EXOGENEITY AND THE ORDER OF COINTEGRATION IN UK MONEY DEMAND DATA

by

Søren Johansen[1]

Abstract This paper discusses autoregressive models allowing for processes integrated of order 2, and the various types of cointegration that can occur. A statistical analysis for such models which allows for the determination of the order of integration and the cointegrating ranks is outlined. The notion of weak exogeneity is discussed for I(1) processes. The results are illustrated by the UK money data of Hendry and Ericsson (1991).

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0 Introduction

Many macro variables show trending behavior over time. This phenomenon is in econometrics modeled by non–stationary time series, and this paper discusses autoregressive Gaussian models for p–dimensional systems of economic variables. Such a time series X_t is called I(1) if the difference $\Delta X_t = X_t - X_{t-1}$ is stationary, while X_t is non–stationary. For a p–dimensional system of I(1) processes, it can occur that a linear combination $\nu' X_t$ is stationary. If this is the case the variables are called cointegrated, and the stationary relation is called a cointegrating relation and ν the cointegrating vector. An alternative formulation is that the common non–stationary forces, the so–called common trends, can be eliminated by considering the linear combination $\nu' X_t$.

Some time series like the log of prices (p) have the property that even the inflation rate Δp is non–stationary, whereas the second difference $\Delta^2 p$ is stationary. Such a variable is called I(2), and we shall show by a statistical analysis how one can analyze processes that may be I(2).

For I(2) variables various types of cointegration can occur. First of all linear combinations of I(2) variables may be I(1) or even I(0), but it is also possible that there are linear I(1) combinations that cointegrate with the difference of the process, which is an I(1) process.

The concept of cointegration was introduced by Granger (1981) and is used in econometrics to discuss long–run economic relations. This definition allows the question of existence of long–run economic relations to be discussed from a statistical point of view which is what we shall do in the present paper.

A very important consequence of the basic definition of cointegration as a statistical concept is that the cointegrating properties of a multivariate time series can be analyzed from the reduced form of the model, even if they gain their importance only when interpreted in a suitable structural model.

It was shown in Johansen (1988), Johansen and Juselius (1990), and Ahn and Reinsel (1990) how one can make inference on the number of cointegrating relations and how one can test hypotheses about the coefficients of the cointegrating relations, such that economic questions and hypotheses can be tested against the data.

There are in particular two questions that we shall be concerned with in this paper, namely the order of integration of the variables and the concept of weak exogeneity.

The statistical analysis of a system of variables is somewhat involved and is sometimes replaced by the analysis of a partial system, thereby reducing the dimensionality of the system whose properties need to be modeled explicitly. The concept of weak exogeneity, see Engle, Hendry and Richard (1983), was introduced to justify considering some variables given (exogenous) in the analysis of other (endogenous) variables. It is important to emphasize that weak exogeneity is also a statistical concept, and as such can be tested against the data.

With this background the purpose of the present paper is to show by example how one can test for weak exogeneity, and investigate the order of non–stationarity of the processes.

1 Error correction models, cointegration and the I(1) analysis

This section contains a reformulation of the VAR model as a reduced form error correction model, and a discussion of the hypothesis of cointegration and its consequences for the process formulated as a representation theorem for I(1) variables. The statistical analysis based upon the likelihood function is described and the results are illustrated using the money demand data for UK. We show how the cointegrating rank can be determined and how some hypotheses on the coefficients of the long–run relations can be tested.

1.1 The reduced form error correction model and the representation of I(1) processes

The p–dimensional vector autoregressive process X_t is defined by the equations

$$X_t = \Sigma_1^k \Pi_i X_{t-i} + \mu + \epsilon_t, \qquad t = 1,\ldots, T, \tag{1}$$

where $X_{-k+1},\ldots,X_0$ are fixed and $\{\epsilon_t,\ t = 1,2,\ldots\}$ is a sequence of independent Gaussian variables with mean zero and covariance matrix Ω. The parameters are the p×p matrices $\Pi_1,\ldots,\Pi_k$, the covariance matrix Ω together with the p–dimensional vector μ. The model can be written as a reduced form error correction model

$$\Delta X_t = \Pi X_{t-1} + \Sigma_1^{k-1}\Gamma_i \Delta X_{t-i} + \mu + \epsilon_t, \quad t = 1,\ldots,T, \tag{2}$$

where $\Pi = \Sigma_1^k \Pi_i - I$, $\Gamma_i = -\Sigma_{i+1}^k \Pi_j$, $i = 1,\ldots,k-1$, or in anticipation of the later analysis of the I(2) model we can write it as

$$\Delta^2 X_t = \Pi X_{t-2} + \Gamma\Delta X_{t-1} + \Sigma_1^{k-2}\Gamma_i^* \Delta^2 X_{t-i} + \mu + \epsilon_t, \quad t = 1,\ldots,T, \tag{3}$$

where $\Gamma = \Sigma_1^{k-1}\Gamma_i - I + \Pi$ and $\Gamma_i^* = -\Sigma_{i+1}^{k-2}\Gamma_j$, $\qquad i = 1,\ldots,k-2.$

The advantage of this reformulation is that the hypothesis of cointegration can be formulated entirely as a restriction on the matrix Π, leaving the other parameters unrestricted.

For r = 0,1, ... ,p, the hypothesis of at most r cointegrating vectors is defined as the reduced rank condition

$$H_r\colon\ \Pi = \alpha\beta' \tag{4}$$

where α and β are p×r matrices. Thus H_0 specifies that $\Pi = 0$, and H_p that Π is unrestricted. If rank(α) = rank(β) = r, and if

$$\text{rank}\{\alpha_\perp'\Gamma\beta_\perp\} = p-r, \tag{5}$$

then the process X_t generated by model (1), or equivalently (2), is non–stationary but the differences are stationary, that is, X_t is an I(1) process. We have used the notation $\alpha_\perp$ for a p×(p–r) matrix of full rank such that $\alpha'\alpha_\perp = 0$, and hence rank$(\alpha,\alpha_\perp) = p$. In case r = 0, that is, when r, α and β are all zero, we define $\alpha_\perp = \beta_\perp = I$. In this case (2) becomes a model for ΔX_t, and the full rank condition (5) reduces to the requirement that ΔX_t be an invertible stationary process which excludes cointegration between levels.

Below we multiply the equations by the full rank matrix $(\alpha,\alpha_\perp)$ and transform the variables using the full rank matrix $(\beta,\beta_\perp)$. It is a property of the system (1) with conditions (4) and (5) that the linear combinations $\beta' X_t$, the cointegrating relations, are stationary. The fundamental paper by Engle and Granger (1987) has the basic definitions and results about this type of non–stationary processes, which can be summarized in Granger's representation theorem which describes the solution of the equation (1) under condition (4) and (5)

$$X_t = C\sum_{i=1}^{t}\epsilon_i + C_1(L)\epsilon_t + \tau_0 + \tau_1 t,$$

where $C = -\beta_\perp(\alpha_\perp'\Gamma\beta_\perp)^{-1}\alpha_\perp'$ so that $\beta' C = C\alpha = 0$. The trend is determined by $\tau_1 = C\mu$, so that the trend vanishes if $\alpha_\perp'\mu = 0$.

1.2 The statistical analysis of the I(1) model

The statistical analysis of model (2) under the restriction of reduced rank of the matrix Π can be performed by reduced rank regression as introduced by Anderson (1951), see also Johansen (1988) for the application to non–stationary processes and Ahn and Reinsel (1988) for the application to stationary processes. The variables ΔX_t and X_{t-1} are regressed on the lagged values $\Delta X_{t-1},\ldots,\Delta X_{t-k+1}$ and 1 to form residuals R_{0t} and R_{1t}, and residual product moment matrices

$$S_{ij} = T^{-1}\Sigma_1^T R_{it}R_{jt}', \qquad i,j = 0,1.$$

The cointegrating relations are then estimated as the eigenvectors corresponding to the r largest eigenvalues of the equation

$$|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}| = 0. \tag{6}$$

The likelihood ratio test statistic of the hypothesis H_r in H_p is given by the so–called trace statistic:

$$Q_r = -T\Sigma_{r+1}^{p}\ln(1-\hat{\lambda}_i). \tag{7}$$

Under the assumption that the number of cointegrating relations is r, and that the coefficient $\alpha_\perp'\mu \neq 0$, such that there is a linear trend in the data, the limit distribution, which only depends on the degrees of freedom p–r, is non–standard and tabulated by simulation in Johansen and Juselius (1990) Table A1. The hypothesis H_r^* that the trend is absent $(\alpha_\perp'\mu = 0)$ can be analyzed by another reduced rank regression, and the test statistic Q_r^* of H_r^* in H_p has under H_r^* a limit distribution given by Table A3 in Johansen and Juselius (1990).

1.3 An illustration of the I(1) analysis by the UK money demand data

To illustrate the results of the I(1) analysis we analyze the data given in Hendry and Ericsson (1991) on the UK money demand, and discussed in Ericsson, Campos and Tran (1991) and Hendry and Mizon (1990). The data consists of four variables: The measure of money M, is nominal money M1. The income measure is denoted by INC, and is given by constant price Total Final Expenditure at 1985 prices. The price measure P, is the implicit deflator of TFE, and as a measure of the opportunity cost of holding money we use R^*. Here R^* is defined as the three month local authority interest rate (R3) less the learning adjusted retail sight–deposit interest rate (Rra). The data are quarterly seasonally adjusted from 1963(1) to 1989(2) and the first six observations are used as initial values in order to fit an autoregressive model with 5 lags. This leaves a total of 100 effective observations. The data is carefully discussed in the above references and the present analysis is a supplement to the previous analysis with respect to the question of weak exogeneity and the question of the order of integration of the variables.

The data are transformed logarithmically into m_t, inc_t and p_t, whereas the interest rate R_t^* is kept untransformed. Figure 1 shows the non–stationarity of the variables m and p, and Figure 2 shows the differences of m and p.

Figure 1 and 2 about here

The results of the initial cointegration analysis from model (2) with 5 lags reproduce the results in Ericsson, Campos and Tran (1991).

Table 1 here

The determination of the cointegrating rank is made difficult by the many hypotheses that can be formulated, and by the non–standard limit distributions. As mentioned in section 1.2, the limit distribution of Q_r, see (7), depends on the presence or absence of the trend. The distribution of Q_r, if in fact the trend is absent, is given by Table A2 in Johansen and Juselius (1990) and has broader tails than that given by Table A1. Thus if one wants to make sure that the size of the test based upon Q_r has the correct value for all parameter points in H_r, one should apply the quantiles in Table A2. This procedure increases the quantiles considerably for small degrees of freedom, and instead another procedure is suggested, see Johansen (1991a), based upon an idea of Pantula (1989). The idea is to use not one test statistic to reject H_r but two, namely Q_r compared to its quantile c_r given by Table A1 and Q_r^* compared to its quantile c_r^* given by Table A3. Hence H_r is rejected if $H_0, \ldots, H_{r-1}$ are rejected and if further

$$Q_r^* > c_r^* \text{ and } Q_r > c_r \,.$$

This procedure guarantees that the asymptotic size of the test is correct for all parameter values in $H_r \setminus H_r^*$.

In the example we have concluded on the basis of Figure 1 that a trend is needed to describe the data. Thus we have accepted that Q_r^* is larger than its quantile, and the rest of the analysis only requires the quantiles as given by Table A1. The cointegrating rank can be formally estimated as the smallest r which is not rejected at a given level of significance. In the present example, see Table 1, we can clearly reject $r = 0$, since the test statistic is 77.54 and the quantile is only 47.18. The hypothesis H_1 of $r \leq 1$ is a borderline case since the statistic 30.50 corresponds roughly to the 95% quantile in the asymptotic distribution. The hypothesis H_2 can be accepted. If we decide to accept H_1 we can work with $r = 1$ in the following, which we shall do, but the evidence for a choice between $r = 1$ and $r = 2$ is not very strong. Having decided that $r = 1$ the estimate of β is given as the first column of the eigenvectors in Table 1, and the estimate of α is the first column of the adjustment coefficients in Table 1. Note that if we had chosen $r = 2$, the estimates of α and β are given as the first two columns, thus it is quite easy, once the eigenvalue problem has been solved, to do the analysis for various values of r.

The solution of the eigenvalue problem (6) constructs the eigenvectors as new regressors in model (2). They are normalized such that the eigenvalues λ_i measure the size of the adjustment coefficients: $\lambda_i = \alpha_i' S_{00}^{-1} \alpha_i$. Thus the test that $r = 1$ is really a test that $\lambda_2 = \lambda_3 = \lambda_4 = 0$, whereas $\lambda_1 > 0$, or equivalently that the α's in the last three columns are insignificantly small.

It is quite clear that the first cointegrating relation, the first column of the eigenvectors, shows homogeneity of the price and income variables and this can be formulated as a test on the coefficients β of the form $K'\beta = 0$ where K contains the two vectors (1,1,0,0) and (1,0,1,0). This type of hypothesis was analyzed in Johansen and Juselius (1990) and there it was shown, that under such a restriction the statistical analysis was still given by a reduced rank regression, and that the test statistic was a comparison of the eigenvalues from (6) by means of the statistic

$$T\Sigma_1^r \ln\{(1-\tilde{\lambda}_i)/(1-\hat{\lambda}_i)\} = 1.02,$$

which in this case is asymptotically χ^2 with 2 degrees of freedom. Here $\hat{\lambda}_i(\tilde{\lambda}_i)$ is calculated without (with) the restrictions on β. Since $\chi^2_{95\%}(2) = 5.99$ the test statistic is seen not to be significant and the analysis of the full system can be summarized from the point of view of the long–run relation by

$$m_t^{lr} = p_t + inc_t \; - 7.01R_t^*,$$

which represents the optimal estimate of the long–run economic relation between the variables provided model (2) with condition (4) and $r = 1$ is maintained. The coefficients in the relation are found as the first column of the eigenvectors estimated under the restriction $K'\beta = 0$.

2 Partial systems and weak exogeneity

This section contains a discussion of the concepts of weak exogeneity and partial or conditional models. The hypothesis of weak exogeneity for the long–run parameters is formulated as a parametric restriction on the adjustment coefficients and the procedure is illustrated by the UK money demand data.

2.1 Weak exogeneity and the efficiency of partial models

An advantage of the vector autoregressive formulation is that one can formulate a partial system as a conditional model and discuss its properties. That is, despite ones interest in modeling only the equations of some of the variables given the others, the stochastic properties of the conditioning variables are well defined in the VAR model.

Consider therefore the autoregressive model (2) under the hypothesis of cointegration H_r, see (4). Let the process X_t be decomposed into the variables Y_t and Z_t of dimension p_y and p_z respectively, where $p = p_y + p_z$, and let α, $\Gamma_1, \dots, \Gamma_{k-1}$, μ, ϵ_t and Ω be decomposed correspondingly. Model (2) can be decomposed into the conditional model for Y_t given Z_t:

$$\Delta Y_t = \omega\Delta Z_t+(\alpha_y-\omega\alpha_z)\beta'X_{t-1}+\Sigma_1^{k-1}(\Gamma_{yi}-\omega\Gamma_{zi})\Delta X_{t-i}+\mu_y-\omega\mu_z+\epsilon_{yt}-\omega\epsilon_{zt}, \tag{8}$$

and the marginal model of Z_t:

$$\Delta Z_t = \alpha_z\beta'X_{t-1} + \Sigma_1^{k-1}\Gamma_{zi}\Delta X_{t-i} + \mu_z + \epsilon_{zt}, \tag{9}$$

where $\omega = \Omega_{yz}\Omega_{zz}^{-1}$.

Note that all the cointegrating relations $\beta'X_{t-1}$ enter into the marginal as well as the conditional model, and that the conditional model has new adjustment coefficients

$\alpha_y - \omega\alpha_z$ depending on the covariance matrix of the errors and all the adjustment coefficients. In general the parameters of the marginal and the conditional system are interrelated which means that full system analysis is needed to draw efficient inference about the parameters.

There is, however, a very special case in which the partial model (8) contains as much information as the full system about the cointegrating relations and the adjustment coefficients, and where analysis of the partial model is efficient. This is when Z_t is weakly exogenous for α and β, see Engle, Hendry and Richard (1983).

The variable Z_t is said to be weakly exogenous for the parameters of interest, if

Condition 1 The parameters of interest are functions of the parameters in the conditional model.

Condition 2 The parameters in the conditional model and the parameters in the marginal model are variation free, that is, they do not have any joint restrictions.

It can be shown that if we define the parameters of interest in model (2) to be all the parameters of β, then weak exogeneity of Z_t with respect to β equivalent to the condition that $\alpha_z = 0$, that is, the rows of α corresponding to the z–equations are zero, and the models (8) and (9) reduce to

$$\Delta Y_t = \omega\Delta Z_t + \alpha_y\beta' X_{t-1} + \Sigma_1^{k-1}(\Gamma_{yi} - \omega\Gamma_{zi})\Delta X_{t-i} + \mu_y - \omega\mu_z + \epsilon_{yt} - \omega\epsilon_{zt}, \qquad (10)$$

and

$$\Delta Z_t = \Sigma_1^{k-1}\Gamma_{zi}\Delta X_{t-i} + \mu_z + \epsilon_{zt}. \qquad (11)$$

In this case β and the remaining adjustment coefficients α_y enter only in the partial model (10), and the properties of the Gaussian distribution show that the parameters in the models (10) and (11) are variation free, see Johansen and Juselius (1990) for a discussion of the results, and Johansen (1990c) and Boswijk (1990) for a fuller discussion of partial systems. Note that equation (11) contains as well ΔZ_{t-i} as ΔY_{t-i}. If also the coefficients of ΔY_{t-i} are zero, or in other words Y_t does not Granger cause Z_t, then Z_t is said to be strongly exogenous for β. Thus weak exogeneity means that ΔZ_t does not react to disequilibrium errors, but may still react to lagged changes of Y_t, and strong exogeneity implies that ΔZ_t does not react to the lagged Y_t, whether Y is in changes or levels.

It should be pointed out that the notion of weak exogeneity depends on an explicit choice of parameters of interest. If for instance $p_y = 1$, and if we define the parameters of interest to be the cointegrating vector in the conditional model $\beta_y' = (\alpha_y - \omega\alpha_z)\beta'$, then condition 1 above is certainly satisfied. If we then decompose β into β_y and β_z and write $\alpha_z\beta' = \alpha_{zy}\beta_y' + \alpha_{zz}\beta_z'$, then the parameters of the conditional and marginal models become variation free if only $\alpha_{zy} = 0$. Thus for this choice of parameters of interest, Z_t is weakly exogenous if $\alpha_{zy} = 0$. This situation occurs in that analysis of Hendry and Mizon (1990).

The statistical analysis of (10) consists of a reduced rank regression if $p_y > r$, and an ordinary regression if $p_y = r$. In particular, if $p_y = r = 1$, the analysis of (10) reduces to the well–known single–equation analysis. It is seen that this analysis is efficient if the remaining variables are weakly exogenous for β, and if there is only one cointegrating relation.

Modeling and analyzing the partial system with $p_y = r$ is therefore simpler and easier to interpret, but the usefulness is limited by the fact that more assumptions need to be made, thus for instance one has to assume weak exogeneity and $p_y = r$, which it is easy to assume but difficult to check efficiently without doing a full system analysis.

Thus the hypothesis of weak exogeneity of Z_t for α and β is formulated as

$$H: \ \alpha_z = 0.$$

This hypothesis is a linear restriction on α and is discussed in Johansen and Juselius (1990), where it was shown that under the hypothesis H the maximum likelihood estimation of the parameters could be performed by reduced rank regression, and that the test of H in H_r consists in comparing the eigenvalues $\hat{\lambda}_i(\tilde{\lambda}_i)$ calculated without (with) the restriction. The test statistic is

$$T\Sigma_1^r \ln\{(1-\tilde{\lambda}_i)/(1-\hat{\lambda}_i)\}, \tag{12}$$

which is asymptotically distributed as $\chi^2(rp_z)$.

The test requires that the partial model be embedded in the full model, and in fact comes from an analysis of the full model. For the present example where only 4 variables enter the system, the analysis of the full model is relatively simple. For systems with 25 variables the full system analysis is more difficult to perform, and it is suggested to make a simplified analysis by assuming that $p_y \geq r$ and analyze the partial model by reduced rank regression. The weak exogeneity can then be tested by an F–test in the marginal model (11), as the hypothesis that the coefficients to the added regressor $\hat{\beta}'X_{t-1}$ is zero, see Johansen (1990c).

2.2 An illustration of the test for weak exogeneity

In the example we have calculated the statistic (12) to test the weak exogeneity of each of the variables m, p, inc, and R^*, in the hope that one can justify the analysis of a single equation.

Table 2 here

It is seen that we can safely assume that inc and R^* are weakly exogenous for the long–run parameters, but it seems that the equation for p may contain information about the cointegrating relation and a single equation analysis is going to miss this. The test of the hypothesis that p, inc, and R^* are all weakly exogenous for the long–run parameters, that is, that $\alpha_p = \alpha_{inc} = \alpha_{R^*} = 0$, can be performed by the same procedure and it is found that the test statistics is 4.85 which should be compared with a $\chi^2_{95\%}(3) = 7.81$. This is clearly not significant, and it follows that by testing all hypotheses simultaneously one can hide the information in the second equation. There is, however, not strong evidence against weak exogeneity of (p, inc, R^*) but the individual tests indicate that there may be some information in the equation for p and as a consequence it may not be efficient to analyze a single equation to estimate the parameters in β.

The purpose of reporting the single equation for Δm_t given Δp_t, Δinc_t, ΔR_t^* and the lags of all variables is to obtain a single equation describing the dynamics of money. This single equation can of course be derived from the full system, and is given by (8) for $Y_t = m_t$ and $Z_t = (p_t, inc_t, R_t^*)$. For given value of $\beta = \hat{\beta}$ the equation can then be further reduced by conventional F–tests in order to decrease the number of parameters needed in the equation, see Ericsson, Campos and Tran (1991) or Hendry and Mizon (1990) for details.

It should be emphasized that the difficulty with the single–equation approach lies in the estimation of β. The estimator of β is consistent, but the asymptotic distribution of the estimator does not permit the use of the usual χ^2 distribution, unless there is weak exogeneity. This problem is also discussed in Phillips (1991).

Thus it is suggested to keep the full system analysis for inference on β, and once the relevant hypotheses on β have been tested, one can go to the single equation estimation in which one can interpret and make the usual inference on the remaining parameters, keeping β fixed.

3 *Formulation and analysis of a model for I(2) variables.*

Section 3 contains results for I(2) processes that parallel the results for I(1) processes. We formulate conditions on the parameters of the VAR model for the process to be I(2) and discuss the various types of cointegration that can occur. The properties of the process are summarized in a representation for I(2) processes. A statistical analysis is suggested which consists of an analysis of model (2) with reduced rank of Π, followed by a reduced rank regression of an equation derived for the differenced data. The money demand data is used to illustrate the determination of the cointegrating ranks and the long–run economic relation is estimated for the I(2) system.

3.1 Cointegration in the I(2) model and the representation of I(2) processes

Consider again model (2) under the condition of reduced rank (4). If condition (5) fails and the matrix $\alpha_\perp'\Gamma\beta_\perp$ has reduced rank:

$$\alpha_\perp'\Gamma\beta_\perp = \varphi\eta', \tag{13}$$

where φ and η are (p–r)xs of rank s, and if a further full rank condition is satisfied, then the process X_t is I(2). In the special case r = 0, so that $\alpha = \beta = 0$ and $\alpha_\perp = \beta_\perp = I$, condition (13) reduces to the condition that the impact matrix for the process ΔX_t has reduced rank, allowing ΔX_t to be I(1) and hence X_t to be I(2). In any case one obtains, see Johansen (1990b), that the properties of the process can be summarized by the representation

$$X_t = C_2 \sum_{j=1}^{t} \sum_{i=1}^{j} \epsilon_i + C_1 \sum_{i=1}^{t} \epsilon_i + C_2(L)\epsilon_t + \tau_0 + \tau_1 t + \tfrac{1}{2}\tau_2 t(t+1).$$

The matrices C_2 and C_1 determine the cointegration properties of the process, and since $\beta' C_2 = 0$, but $\beta' C_1 \neq 0$ it is seen that $\beta' X_t$ is not stationary in general, but only I(1). In order to make it stationary one needs to bring in the differences in the form $\eta_\perp'\beta_\perp'\Delta X_t$, and it can be shown that

$$\beta' X_t + \kappa\beta_\perp^{2\prime}\Delta X_t \tag{14}$$

is a stationary process, where $\beta_\perp^2 = \beta_\perp\eta_\perp$, and $\kappa = (\alpha'\alpha)^{-1}\alpha'\Gamma\beta_\perp^2(\beta_\perp^{2\prime}\beta_\perp^2)^{-1}$. The vectors $\beta_\perp^1 = \beta_\perp(\beta_\perp'\beta_\perp)^{-1}\eta$ determine other combinations that reduce the order from 2 to 1, but they do not cointegrate with the differences. Similarly we define $\alpha_\perp^1 = \alpha_\perp'(\alpha_\perp\alpha_\perp)^{-1}\varphi$, and $\alpha_\perp^2 = \alpha_\perp\varphi_\perp$. With this notation we can express

$$C_2 = \beta_\perp^2\tau\alpha_\perp^{2\prime}, \tag{15}$$

for some (p–r)x(p–r) matrix τ of full rank.

Thus the p components of X_t are split into three sets of dimensions r, s, and p–r–s respectively. The p×(r+s) matrix $(\beta,\beta_\perp^1)$ represents all the possible cointegrating relations, in the sense that $(\beta,\beta_\perp^1)'X_t$ is either I(1) or I(0), whereas $\beta_\perp^{2\prime}X_t$ is an I(2) process that does not cointegrate. The process $\beta_\perp^{1\prime}X_t$ is an I(1) process which does not cointegrate, and finally $\beta'X_t$ cointegrate with the I(1) process $\beta_\perp^{2\prime}\Delta X_t$ and hence is in general an I(1) process. Finally if ξ is a matrix such that $\xi'\kappa = 0$, then $\xi'\beta'X_t$ is stationary, see (14). The expression (15) for C_2 in terms of $\beta_\perp^2$ and $\alpha_\perp^2$ shows that the cumulated shocks $\sum_{j=1}^{t}\sum_{i=1}^{j}\epsilon_i$ enter the variables through the linear combinations $\alpha_\perp^2$, and that they are distributed among the variables through the coefficients $\beta_\perp^2$. This interpretation will be useful in the discussion of the example below.

3.2 The statistical analysis of the I(2) model

The hypothesis $H_{r,s}$ is defined by conditions (4) and (13) with α and β of full rank r and φ and η of dimensions (p–r)×s. The likelihood function is easily described for the I(2) model due to the Gaussian errors. The analysis, however, does not lead to explicit solutions. The problem is of course how to estimate the parameters α and β at the same time as the matrices φ and η since the second reduced rank condition (13) involves all parameters.

Instead another procedure is suggested, which consists of first analyzing model (2), that is the VAR model, with reduced rank of Π, but without the restriction (13), and then perform another reduced rank regression on a derived equation for the differenced process. The following procedure turns out to yield valid inference, see Johansen (1991b).

A. *Perform a reduced rank regression of*

$$\Delta X_t \text{ on } X_{t-1} \text{ corrected for } \Delta X_{t-1},\ldots,\Delta X_{t-k+1} \text{ and } 1.$$

This determines $\hat{r}$, $\hat{\alpha}$, *and* $\hat{\beta}$.

B. *Perform a reduced rank regression of*

$$\hat{\alpha}'_{\perp}\Delta^2 X_t \text{ on } \hat{\beta}'_{\perp}\Delta X_{t-1} \text{ corrected for } \Delta^2 X_{t-1},\ldots,\Delta^2 X_{t-k+2}, 1 \text{ and } \hat{\beta}'\Delta X_{t-1}.$$

This determines $\hat{s}$, $\hat{\varphi}$, *and* $\hat{\eta}$.

To see that this analysis is relevant consider the equation given by (3) but multiplied by $\alpha'_{\perp}$:

$$\alpha'_{\perp}\Delta^2 X_t = \alpha'_{\perp}\Gamma\Delta X_{t-1} + \Sigma_1^{k-2}\Gamma_i^*\Delta^2 X_{t-i} + \alpha'_{\perp}\mu + \alpha'_{\perp}\epsilon_t. \tag{16}$$

This is now a reduced form error correction model for the differences.

The decomposition

$$\alpha'_{\perp}\Gamma = \alpha'_{\perp}\Gamma\{\beta(\beta'\beta)^{-1}\beta' + \beta_{\perp}(\beta'_{\perp}\beta_{\perp})^{-1}\beta'_{\perp}\} = \alpha'_{\perp}\Gamma\beta(\beta'\beta)^{-1}\beta' + \varphi\eta'(\beta'_{\perp}\beta_{\perp})^{-1}\beta'_{\perp}$$

introduces the second reduced rank condition (13) explicitly into the equation and shows that if we know α and β we can estimate equation (16) by the above mentioned reduced rank regression.

The equation (16) can be given the interpretation as an analysis of the common trends. Granger defines $\alpha'_{\perp}X_t$ as the common trends since the equation for $\alpha_{\perp}$ does not contain any term corresponding to the disequilibrium error. Thus the second step in the analysis is a reduced rank analysis of the common trends.

The reduced rank analysis of (16) reduces to the solution of an eigenvalue problem like (6) giving eigenvalues ρ_i, $i = 1,\ldots,p-r$ and the test statistic for the hypothesis $H_{r,s}$ in H_r is given by

$$Q_{r,s} = -T\Sigma_{s+1}^{p-r}\ln(1-\rho_i) \tag{17}$$

It can be shown, see Johansen (1991b), that the estimators of β and φ are asymptotically mixed Gaussian allowing for usual χ^2 inference, and that $Q_{r,s}$ has the same limit distribution as Q_r, but with p–r–s degrees of freedom. Thus both Q_r and $Q_{r,s}$ have a distribution determined by the deficiency of the matrix being tested for reduced rank.

Another important consequence of the limit results is that the tests performed on the coefficients α and β in the I(1) analysis remain valid in the I(2) analysis, but of course the parameters and hence the hypotheses have a different interpretation. Thus the test that m, p, and inc have coefficients proportional to (1,–1,–1) still holds in the presence of I(2) variables, but for a relation that is I(1). One can also test $\alpha_z = 0$ by a χ^2 test, but the restriction (13) ruins the variation independence of the parameters and hence the interpretation of the hypothesis as a hypothesis of weak exogeneity is not valid. Thus the formal properties of the test remain valid, but the interpretation has changed in the I(2) model.

3.3 An illustrative example

The nominal variables p_t and m_t might well be I(2) instead of I(1) processes. Graphs of the data in differences, see Figure 2, show that indeed the differences could be described by an I(1) process, and we shall here show how one can analyze this phenomenon using the methods of Johansen (1991b).

By repeated application of reduced rank regression we can estimate all parameters in the I(2) model and we demonstrate below how to determine the ranks r and s.

For the UK data we find the results in Table 3 where the test statistics $Q_{r,s}$ and Q_r are given as functions of the degrees of freedom p–r and p–r–s to facilitate the comparison with the quantiles.

Table 3 here.

The value of r is determined by reading the Q_r column from top to bottom and comparing the observed value with the quantile for p–r degrees of freedom from Table A1 in Johansen and Juselius (1990) derived under the assumption that there is a linear trend in the data. Once the value of $\hat{r}$ has been determined one reads the table from left to right in the row with $r = \hat{r}$, and compares with the quantiles with p–r–s degrees of freedom from before, but now listed in the second row from below for ease of reference. This again requires that we accept a constant term in model (16).

Thus the determination of r is exactly as in the I(1) analysis, where we have chosen $\hat{r} = 1$. Reading the row $r = 1$ from left to right we first test the hypothesis $H_{1,0}$ that $s = 0$ by the test statistic 62.32 as compared to the quantile 29.51. Thus $H_{1,0}$ is rejected. Next compare the test statistic $Q_{1,1} = 12.20$ with the quantile 15.20. This hypothesis is accepted, indicating $s = 1$ and that the number of I(2) components is $p-r-s = 2$. Since this is a borderline case we choose to compare also $Q_{1,2} = 4.07$ with its quantile 3.96 which is also a borderline case. Thus there seems to be I(2) variables in the system, but exactly how many is not so clear.

The choice of the 95% quantile is quite arbitrary and the actual distribution of the test statistics is probably not very well approximated by the asymptotic distribution. From economic reasoning it seems plausible that there is no more than one common I(2) variable that drives the other variables, namely one that measures the nominal growth, and this hypothesis is supported by the above analysis of the data and by the graphs that show that only Δp_t and Δm_t can best be described by I(1) processes and $m - p$ by an I(1) process.

We have thus continued the analysis of the data with $r = 1$, $s = 2$ and hence $p-r-s = 1$. We first give the results for the estimates of the matrix Γ in the I(1) model and the I(2) model in Table 4.

Table 4 here

The estimate of Γ in the two models are very similar. This is due to the fact that the reduced rank hypothesis (13) restricts a $(p-r)\times(p-r) = 3\times3$ matrix to have rank $s = 2$. This really loses only 1 degree of freedom, thus the difference between the I(1) model and the I(2) model corresponds to only one parameter restriction. Hence the estimates are rather similar.

If we proceed with the assumption of $r = 1$, $s = 2$ and $p-r-s = 1$, then there is 1 common I(2) trend that drives all the variables. The vectors $\beta' X_t$ is in this case just one linear combination, and it is I(1) and not stationary. It can be made stationary by including the differences, see (14), with coefficients proportional to the vector $\beta_\perp^2 = \beta_\perp \eta_\perp$. Normalized on m_t the stationary relation becomes

$$m_t - 1.04\, p_t - .95\, inc_t + 7.46\, R_t^* + 2.33\, \Delta m_t + 2.71\, \Delta p_t - .30\, \Delta inc_t + .03\, \Delta R_t^*. \quad (18)$$

It must be emphasized that since $(\beta,\beta_\perp^1)'\Delta X_t$ is stationary there are many different relations between $\beta' X_t$ and ΔX_t that are stationary. We shall derive another below see (19) and (20).

Table 5 here

The interpretation of the vectors in Table 5 is not so easy. First of all the vector β is taken from Table 1 with the corresponding adjustment coefficient α. The vector no longer represents a stationary relation. In order to make it stationary it has to be corrected for the differences using $\beta_\perp^{2\prime}\Delta X_t$, see (14). The adjustment coefficients α have the interpretation as the strength of the adjustment to the disequilibrium error defined by (14). The vectors $\beta_\perp^1$ represent I(1) variables which do not cointegrate, and the last column, $\beta_\perp^2$, represents an I(2) variable which can be used to make $\beta' X_t$ stationary as described.

The parameters $\alpha_\perp^2 = \alpha_\perp \varphi_\perp$ are of special interest since the common I(2) trend is given by $\alpha_\perp^{2\prime} X_t$, which in this case is practically equal to the price variable. Thus the price is picked out as the variables that does not react to the disequilibrium error and in this sense it is the common driving I(2) force. The coefficients of $\beta_\perp^2$ from Table 5 show that the I(2) variables are mainly to be found in the variables m and p, see the expression (15) for the coefficients C_2 to the twice cumulated shocks. Since the first two coefficients of $\beta_\perp^2$ are approximately equal, the common I(2) trend can be eliminated by taking the difference $m - p$.

In view of this analysis it seems that by introducing the variable $m_t - p_t$ one can eliminate the I(2) component and then work with the inflation rate $\text{infl}_t = \Delta p_t$ instead of p_t. With the variables $(m_t-p_t, \text{infl}_t, \text{inc}_t, R_t^*)$ we can repeat the I(1) analysis, and find first the results summarized in Table 6.

Table 6 here

It is seen from Table 6 that again there is just one cointegrating vector in the data. The coefficients are about the same as for the first eigenvector of Table 5, but this time with the coefficient of 7.22 to infl_t. Thus it is seen that the inflation rate and the interest rate suffice to reduce the variable $(m-p-y)_t$ to stationarity. For this interpretation to hold we need to check that the reduction did in fact remove the I(2) components. Hence the I(2) analysis is performed for the new system and the results are reported in Table 7.

Table 7 here

From Table 7 it follows that for $r = 1$, the values $s = 0$, 1, and 2 are rejected, corresponding to the choice of $s = p-r = 3$, in which case the matrix in (13) has full rank, such that there are no I(2) variables.

Thus it seems that the I(2) analysis can be avoided by introducing real variables rather than nominal values. It is interesting to compare the cointegrating relation derived from this second system, as given by the first column in Table 6, with the relation (18) derived as stationary from the I(2) analysis. In equation (14) we have have used a vector proportional to $\beta_\perp^2$ in order to achieve stationarity. In reality we can use any vector not orthogonal to $\beta_\perp^2$, that is, any vector not in the space spanned by β and $\beta_\perp^1$. The reason for this is of course that the vectors $\beta'\Delta X_t$ and $\beta_\perp^{1\prime}\Delta X_t$ are already stationary and can be added to (18) without changing the stationarity of the relation. Any vector v can be decomposed into the directions β, $\beta_\perp^1$, and $\beta_\perp^2$, and only the projection in the direction $\beta_\perp^2$ is needed. This projection is given by $\beta_\perp^2(\beta_\perp^{2\prime}\beta_\perp^2)^{-1}\beta_\perp^{2\prime}v$, which shows that we can replace the vector $\beta_\perp^2 = (2.33,2.71,-.30,.03)$ by the vector $(\beta_\perp^{2\prime}\beta_\perp^2)(\beta_\perp^{2\prime}v)^{-1}v$. As an example we can express the relation (18) as a relation between levels and Δp by considering $v = (0,1,0,0)$. This gives

$$m_t - 1.04\ p_t - .95\ inc_t + 7.46\ R_t^* + 4.75\ \Delta p_t. \tag{19}$$

This result from the I(2) analysis of m, p, inc, and R^* is plotted in Figure 1 and can then be compared with the result from the above I(1) analysis of the reduced system which gives

$$m_t - p_t - 1.08\ inc_t + 7.16\ R_t^* + 7.22\ \Delta p_t. \tag{20}$$

These equations are derived from different models, and the differences in the coefficients reflect the statistical variability of the estimates.

It should be pointed out that a sudden jump in an otherwise stationary variable can be mistakenly considered as an indication of a general persistence of the shocks to the variable that is as an indication that the variables are I(1). Thus one should be careful about drawing too strong conclusions from these various ways of looking at the data. What is demonstrated here is that the tools now exist for a statistical analysis, and they have to be combined by careful inspection of residuals, constancy over time of the estimated parameters etc.

4 *Conclusion*

We have illustrated two statistical methods by an analysis of UK money demand. The test of weak exogeneity of the variables p_t, inc_t, and R_t^* requires that the full data vector can be described by an autoregressive model. If this is the case a simple test can be performed. The test is needed as a requirement for the simpler analysis of a single–equation regression to be efficient. Weak exogeneity is devised in order to avoid the investigation of a full system, yet the test requires the modeling of a full system. If one can apply a VAR model for this, the easiest is of course to analyze the full system.

After this has been done one can derive the conditional model that one would like to interpret, in the present case the conditional model for the changes of m_t conditional on p_t, inc_t and R_t^* together with the lags of all variables. This last reduction does not require weak exogeneity. Weak exogeneity is only relevant if one wants to apply the conditional model for the estimation of the long–run parameters.

The presence of I(2) components makes the analysis more difficult, not so much because the method gets very much more involved, but because the interpretation becomes more difficult; what was stationary before is now just I(1), and the differences of the process have to be invoked to produce a stationary relation. The I(2) analysis, however, allows one to identify the common I(2) trends which drive the economy, and which lend themselves to an economic interpretation. The challenge with the I(2) analysis is that it seems to allow for more economic questions to be asked and interpreted.

It is important to note that if one wants to describe this data by an I(2) model there are different choices of the dimensions r and s that are consistent with the data. The values chosen are chosen partly on the basis of the statistical analysis and partly on the basis of what is economically reasonable. We have fitted five lags to a 4 dimensional series which gives 94 parameters and 400 observations. The I(2) model rests on only one restriction of these parameters, and it is important to check to what extend the conclusions depend on the choice of model.

The present analysis is meant as a first illustration of a new technique, where the details have yet to be worked out, and it will take some time until we understand where these methods can be applied with success.

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Table 1

The cointegration analysis of the UK money demand data
The variables are m, p, inc and R^*.

Eigenvalues	.375	.166	.115	.002
Hypotheses	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Trace statistics	77.54	30.50	12.35	.150
95% *Quantiles*	47.18	29.51	15.20	3.96
	Eigenvectors β			
m	1.00	1.22	–.61	1.17
p	–1.04	1.00	.35	–1.05
inc	–.95	–9.82	1.00	–.70
R^*	7.46	–8.11	–3.06	1.00
	Adjustment coefficients α			
Δm	–.150	–.007	–.006	–.001
Δp	.030	–.008	.006	.001
Δinc	.008	.001	.053	–.001
ΔR^*	.031	.001	.031	.003

Table 2

The likelihood ratio statistics for testing weak exogeneity
of each of the variables with respect β.
The asymptotic distribution is $\chi^2(1)$
for which the 95% quantile is 3.84.

m	p	inc	R^*
24.82	4.21	.11	1.89

Table 3

The results of the I(2) analysis of the UK money demand data

r	$Q_{r,s}$				Q_r	tr(95%)	p–r
0	108 . 46 s=0	57 . 97 s=1	14 . 68 s=2	4 . 39 s=3	77.54	47.18	4
1		62 . 32 s=0	12 . 20 s=1	4 . 07 s=2	30.50	29.51	3
2			24 . 89 s=0	5 . 76 s=1	12.35	15.20	2
3				. 29 s=0	.15	3.96	1
tr(95%)	47.18	29.51	15.20	3.96			
p–r–s	4	3	2	1	0		

Table 4

Γ *estimated from the I(1) model*

m	–2.078	.974	–.112	–.397
p	.319	–.224	.297	.100
inc	.486	–.619	–1.861	.327
R^*	.384	–.129	.603	–1.069

Γ *estimated from the I(2) model*

m	–2.037	1.032	–.105	–.379
p	.373	–.148	.306	.123
inc	.500	–.600	–1.859	.333
R^*	.395	–.114	.605	–1.065

Table 5

The estimates of the cointegrating vectors β, and the supplementary vectors $\beta_\perp^1 = \beta_\perp(\beta_\perp'\beta_\perp)^{-1}\eta$ and $\beta_\perp^2 = \beta_\perp\eta_\perp$.

	β	$\beta_\perp^1$		$\beta_\perp^2$
m	1.00	.88	8.24	–.97
p	–1.04	–1.46	–7.41	–1.13
inc	–.95	–6.52	–3.17	.12
R^*	7.46	–1.15	–2.54	–.01.

The estimates of the adjustment coefficients α and the supplementary vectors $\alpha_\perp^1 = \alpha_\perp(\alpha_\perp'\alpha_\perp)^{-1}\varphi$ and $\alpha_\perp^2 = \alpha_\perp\varphi_\perp$

	α	$\alpha_\perp^1$		$\alpha_\perp^2$
Δm	–.150	–.010	.005	.046
Δp	.030	–.042	.000	.253
Δinc	.008	.270	.008	.034
ΔR^*	.031	–.076	.022	–.026

Table 6

The I(1) cointegration analysis of the UK money demand data.
The variables are $m-p$, Δp, inc, and R^*.

The eigenvalues	.386	.128	.050	.009
Hypotheses	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Trace statistics	68.58	19.83	6.09	.95
95% *Quantiles*	47.18	29.51	15.20	3.96
	Eigenvectors β			
$m-p$	1.00	–.08	–1.26	1.33
infl	7.22	1.00	16.07	6.56
inc	–1.08	–.04	1.00	–.13
R^*	7.16	–.79	–7.00	1.00
	Adjustment coefficients α			
$\Delta(m-p)$	–.183	–.034	.002	–.003
Δ*infl*	.023	–.046	–.005	.003
Δ*inc*	.000	.227	–.007	–.001
ΔR^*	.034	.139	.002	.007

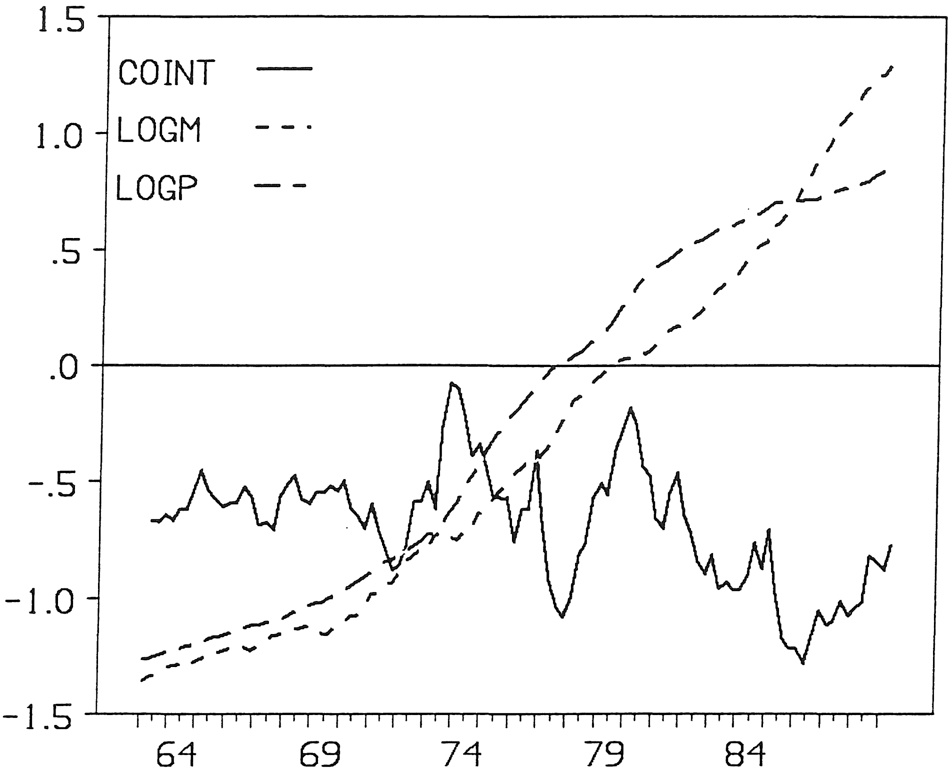
The test for weak exogeneity with respect to α and β.
The 95% quantile of the asymptotic $\chi^2(1)$ distribution is 3.84.

m	*infl*	*inc*	*R**
34.54	3.42	.00	2.25

Table 7

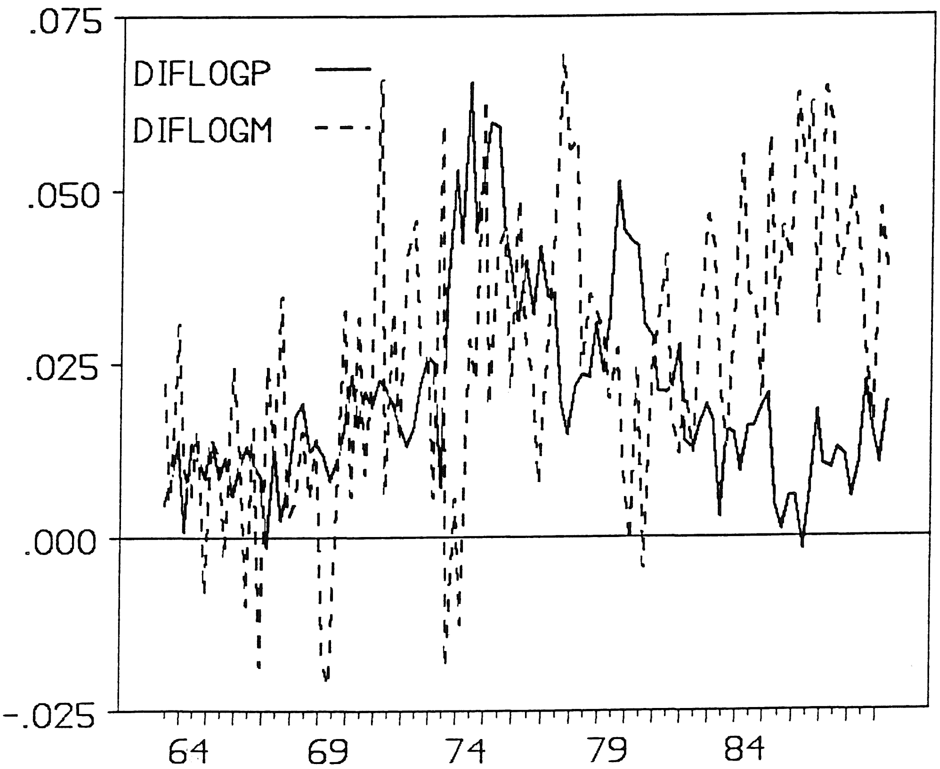
The I(2) analysis of the UK money demand data.
The variables are m−p, infl, inc, and R^*.

r	$Q_{r,s}$				Q_r	tr(95%)	p−r
0	133 . 53 s=0	80 . 62 s=1	39 . 71 s=2	8 . 30 s=3	68.58	47.18	4
1		85 . 16 s=0	37 . 77 s=1	6 . 38 s=2	19.83	29.51	3
2			33 . 40 s=0	1 . 24 s=1	6.09	15.20	2
3				10 . 03 s=0	.95	3.96	1
tr(95%)	47.18	29.51	15.20	3.96			
p−r−s	4	3	2	1	0		



Graphs of the series m_t and p_t together with the disequilibrium error given by

$$coint_t = m_t - 1.04inc_t - .95inc_t + 7.46R_t + 4.75\Delta p_t$$



Graphs of the series Δm_t and Δp_t

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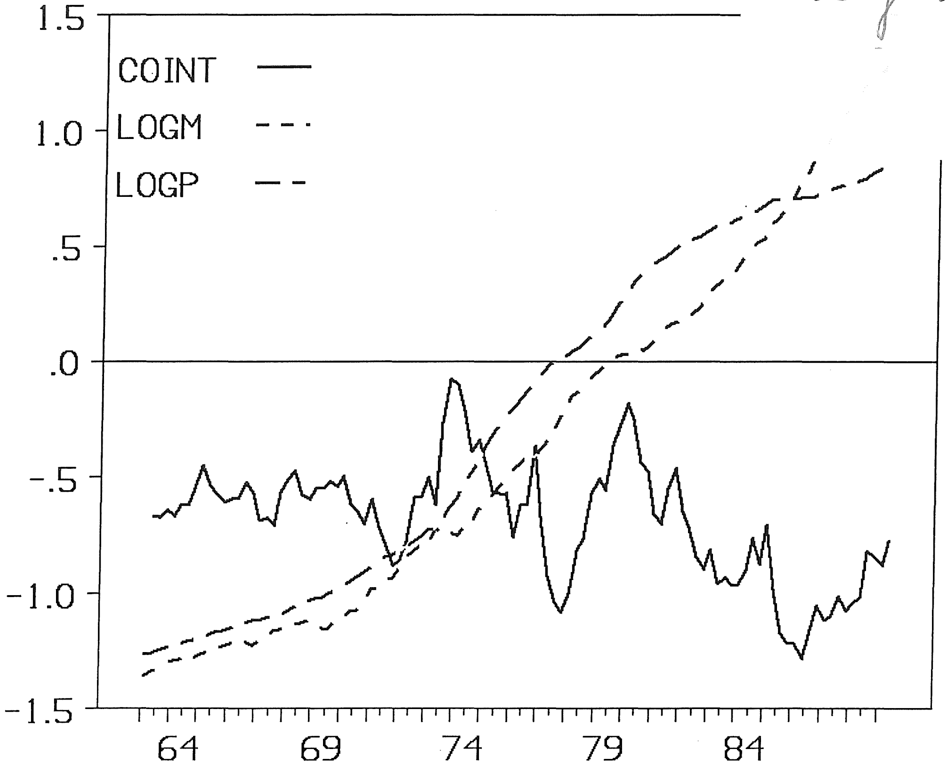

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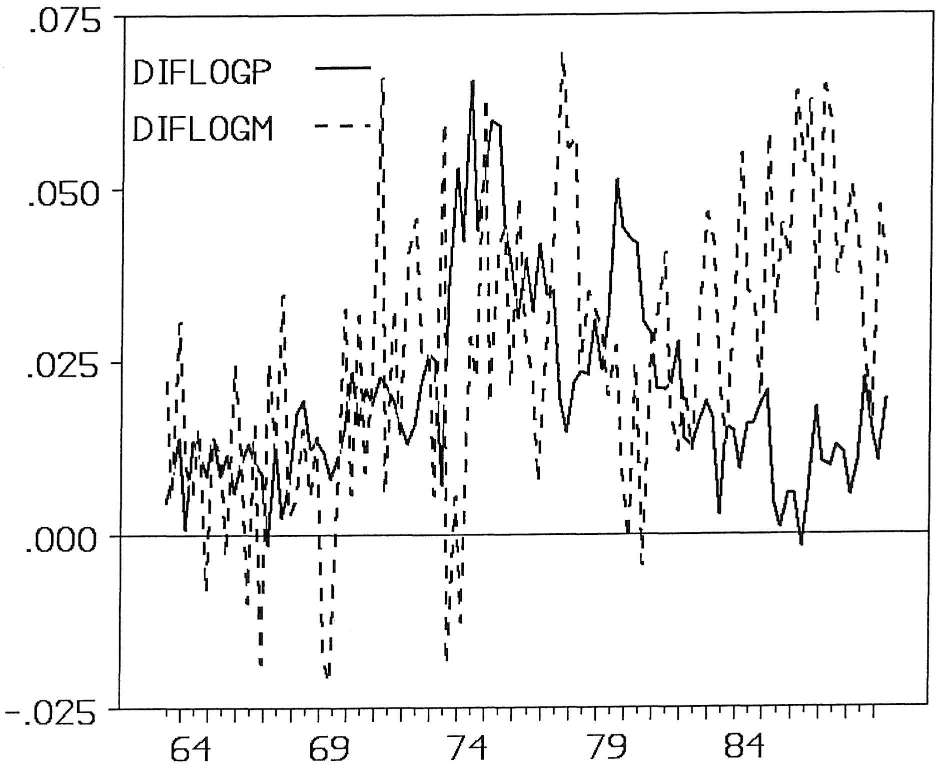

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