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DETERMINATION OF COINTEGRATION RANK IN THE PRESENCE OF A LINEAR TREND

Søren Johansen[1]

Abstract. It is shown how the tables in Johansen and Juselius (1990) can be applied to make inference on the cointegration rank. The reason that inference is difficult is that the limit distribution of the proposed likelihood ratio test statistic depend on which parameter is considered under the null. It is shown how a recent procedure for unit root testing suggested by Pantula (1989) solves the problem. The procedure is illustrated by some published econometric examples.

[1]This paper was written while the author was visiting Department of Statistics, University of Helsinki. The visit was supported by the Danish Social Sciences Research Council acc. nr. 14–5793

I. INTRODUCTION

Consider the vector autoregressive model with Gaussian errors

$$X_t = \Sigma_1^k \Pi_i X_{t-i} + \Psi D_t + \mu + \epsilon_t,\ (t = 1,\dots,T). \tag{1.1}$$

where D_t are the seasonal dummies orthogonalized to the constant term, and $\epsilon_1,\dots,\epsilon_T$ are independent Gaussian variables in p dimensions $N_p(0,\Omega)$. The initial values $X_{-k+1},\dots,X_0$ are kept fixed. The hypothesis of at most r cointegrating relations is formulated as

$$H_r\colon\ I - \Pi_1 - \dots - \Pi_k = \alpha\beta'$$

where α and β are p×r matrices. We denote by $H_r^0 = H_r \setminus H_{r-1}$ the model where the rank of α and β is r, that is, when there are exactly r cointegrating relations. Thus the hypotheses H_r are nested, $H_r \subset H_{r+1}$, and H_r is the union of the non nested models $H_0^0,\dots,H_r^0$. We work under the additional assumption that the variables are I(1) such that X_t is non stationary and ΔX_t is stationary. Grangers result, see Engle and Granger (1987), then states that $\beta' X_t$ is stationary. The presence of the constant term μ implies that in general the process X_t will have a linear trend, see Johansen (1990) for a discussion of this. Here we need the result that the linear trend is absent if $\alpha_\perp'\mu = 0$, and we use this to define the model

$$H_r^*\colon I - \Pi_1 - \dots - \Pi_k = \alpha\beta' \text{ and } \alpha_\perp'\mu = 0. \tag{1.2}$$

The problem that we will discuss in this paper is how to determine jointly both the cointegrating rank r, and whether or not there is a linear trend in the model. That is, which value of r should one use, and if H_r or H_r^* is a better description of the data.

II. THE STATISTICAL ANALYSIS

The analysis of the model H_r is performed by a combination of regression and reduced rank regression, see Johansen (1988) or Reinsel and Ahn (1990), and a detailed analysis of some examples is given in Johansen and Juselius (1990). The analysis will only be discussed very briefly here.

It is convenient to rewrite the equation (1.1) in an error correction form

$$\Delta X_t = \Pi X_{t-1} + \Sigma_1^{k-1}\Gamma_i \Delta X_{t-i} + \Psi D_t + \mu + \epsilon_t. \qquad (2.1)$$

The relation between the parameters $(\Pi,\Gamma_1,...,\Gamma_{k-1})$ and $(\Pi_1,...,\Pi_k)$ is found by identifying coefficients of the lagged levels in the two expressions (1.1) and (2.1).

In model H_r the matrix Π is restricted as $\Pi = \alpha\beta'$, but the parameters $(\alpha,\beta,\Gamma_1,...,\Gamma_{k-1})$ vary independently. Hence the parameters $\Gamma_1,...,\Gamma_{k-1}$ can be eliminated by regressing ΔX_t and X_{t-1} on $\Delta X_{t-1},...,\Delta X_{t-k+1}, D_t$ and 1. This gives residuals R_{0t} and R_{1t}, and residual product moment matrices

$$S_{ij} = T^{-1}\Sigma_1^T R_{it}R_{jt}', \qquad (i,j = 0,1).$$

The remaining analysis of the model H_r can be performed from the equations

$$R_{0t} = \alpha\beta' R_{1t} + \epsilon_t.$$

The estimate of β is determined by reduced rank regression, see Anderson (1951) or Ahn and Reinsel (1988), and is found by solving the eigenvalue problem

$$|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}| = 0, \qquad (2.2)$$

for eigenvalues $\hat{\lambda}_1 > ... > \hat{\lambda}_p$ and eigenvectors $V = (v_1,...,v_p)$. The maximum likelihood estimators are given by

$$\hat{\beta} = (v_1,...,v_r),\ \hat{\alpha} = S_{01}\hat{\beta},\ \text{and}\ \hat{\Omega} = S_{00} - \hat{\alpha}\hat{\alpha}'.$$

Finally the maximized likelihood function is found from

$$L_{max}^{-2/T} = |\hat{\Omega}| = |S_{00}|\Pi_1^r(1-\hat{\lambda}_i).$$

From this it follows that if one wants to test H_r with rank $\leq r$ in H_p with rank $\leq p$, i.e. in the unrestricted VAR model, the likelihood ratio test becomes

$$-2\ln Q(H_r|H_p) = -T\Sigma_{r+1}^{p}\ln(1-\hat{\lambda}_i). \tag{2.3}$$

The asymptotic distribution of the test statistic (2.3) under model H_r^0 and in the presence of a linear trend is given in Johansen (1990) as a functional of Brownian motion that can be expressed as

$$\mathrm{tr}\{\textstyle\int_0^1(dB)F'[\int_0^1 FF'du]^{-1}\int_0^1 F(dB)'\}. \tag{2.4}$$

Here B is a Brownian motion of dimension p–r on the unit interval, and the p–r dimensional process F has the first p–r–1 components equal to $B_i(u) - \int_0^1 B_i(u)du$, and the last component equal to $t - 1/2$. The dimension p–r is called the degrees of freedom for the test statistic. The distribution is tabulated in Johansen and Juselius (1990) as Table A1.

It turns out, however, that if there is no linear trend, that is, if $\alpha_\perp'\mu = 0$, then the limit distribution of (2.3) is different and given by (2.4) with F defined by $B - \int_0^1 B(u)du$. This distribution is tabulated by simulation in Johansen and Juselius (1990) as Table A2. If in fact the number of cointegrating relations is smaller than r, then the distribution of the likelihood ratio test statistic is different. Thus for instance if there are r–1 cointegrating relations the asymptotic distribution of (2.3) is given as the sum of the p–r smallest eigenvectors of the matrix in (2.4) but with B and F of dimension p–r+1. Thus there are many different distributions of the test statistic under the null H_r, and it is this phenomenon that makes the testing procedure somewhat complicated. This problem is discussed in the next section.

The analysis of the model H_r^* is almost the same as above, using the following trick. If $\alpha_\perp'\mu = 0$, then $\mu = \alpha\beta_0$ for some β_0 (r×1). We then note that

$$\alpha\beta'X_{t-1} + \mu = \alpha\beta'X_{t-1} + \alpha\beta_0 = \alpha\beta^{*\prime}X_{t-1}^*,$$

where $X_t^* = (X_t',1)'$ and $\beta^* = (\beta',\beta_0)'$. Thus by appending 1 to the levels we can eliminate the parameters $\Gamma_1,\ldots,\Gamma_{k-1}$, and Ψ by regressing ΔX_t and X_{t-1}^* on $\Delta X_{t-1},\ldots,\Delta X_{t-k+1}$, and D_t. This gives residuals R_{it}^*, i = 0,1, and product moment matrices S_{ij}^*. The solution for β is then to solve the eigenvalue problem (2.2) with S_{ij}^*

replacing S_{ij}. The test statistic for H_r^* in H_p^* is given by (2.3) with the eigenvalues $\hat{\lambda}_i$ replaced by λ_i^*.

The limit distribution of this statistic is given in Johansen (1990) and is shown to be of the form (2.4) but with F of dimension p–r+1, where F is just B with 1 appended as the last coordinate. This distribution is tabulated by simulation in Johansen and Juselius (1990) as Table A.3. Thus the different choices of F reflect the statistical calculations as well as the probabilistic properties of the process.

III. THE TESTING PROCEDURE

In the previous section various results about the limit behaviour of the test statistics were derived. It was shown that the limit distribution depends on the choice of parameter value in the null, either because the linear trend may be absent, or because the number of cointegrating relations is smaller than r.

In calculating the p–values one then has to maximize the probability of extreme deviations of the statistic under the null, and this leads sometimes to unreasonably large p–values.

Recently Pantula (1989) has made an important contribution towards formulating and solving this type of problem in a discussion of unit root testing. The problem has also been treated by Berger and Sinclair (1982) in the context of the Gaussian distribution where the null was a union of linear subspaces. The idea is that instead of calculating just one statistic for the null, a range of statistics is calculated, one for each of the sub hypotheses, that give rise to a different distribution. The null is rejected only if all of them are rejected.

To present this formally consider linearly ordered parameters spaces

$$\Theta_0 \subset \dots \subset \Theta_p,$$

corresponding to a nested system of hypotheses. One can easily extend this to partially ordered systems, as will be illustrated below, but for notational convenience we formulate the results for linearly ordered systems. Typically Θ_r is a very small subset of Θ_{r+1}. We can test Θ_r against Θ_p by a likelihood ratio test, T_r say, and we assume that the limit distibution of T_r depends on which value of θ we consider in Θ_r. We assume, as is the case in the cointegration models, that the limit distribution of T_r is the same for all $\theta \in \Theta_r \setminus \Theta_{r-1}$. This allows the determination of a quantile $c_r(\alpha)$ such that

$$P_\theta\{T_r \geq c_r(\alpha)\} \to \alpha, \qquad (\theta \in \Theta_r \setminus \Theta_{r-1}).$$

We also assume that the power is reasonable, in the sense that

$$P_\theta\{T_r \geq c_r(\alpha)\} \to 1, \quad (\theta \notin \Theta_r), \tag{3.1}$$

where the limit is taken as the number of observations tends to infinity. The immediate solution to solving the problem of different limit distributions under the null is to consider a quantile $c(\alpha)$ such that

$$\max_\theta \lim P_\theta\{T_r \geq c(\alpha)\} = \alpha,$$

and use $\{T_r \geq c(\alpha)\}$ as the critical set. In the applications here, that would often mean that we should evaluate the probabilities under the smallest set Θ_0 thereby emphasizing a very small part of the null hypothesis H_r and increasing the quantile considerably. The idea proposed by the above mentioned authors is to consider a critical set of the form

$$C_r = \{T_0 \geq c_0(\alpha),\ldots,T_r \geq c_r(\alpha)\}, \ (r = 0,1,\ldots,p-1).$$

That is, to reject H_r only if all previous hypotheses have been rejected starting with Θ_0.

The properties of this procedure are easily derived: It is clear that for $\theta \in \Theta_r$ we must have $\theta \in \Theta_i \setminus \Theta_{i-1}$ for some $i = 0,\ldots,r$. Hence

$$P_\theta(C_r) \leq P_\theta\{T_i \geq c_i(\alpha)\} \to \alpha, \ (\theta \in \Theta_i \setminus \Theta_{i-1}),$$

shows that the asymptotic size is always $\lesssim \alpha$.

Now take $\theta \in \Theta_r \setminus \Theta_{r-1}$, which is the majority of the points in the null hypothesis Θ_r. In this case θ is outside $\Theta_0,\dots,\Theta_{r-1}$ and the power property (3.3) shows that

$$P_\theta(C_r) \to \lim P_\theta\{T_r \geq c_r(\alpha)\} = \alpha, \quad \theta \in \Theta_r \setminus \Theta_{r-1}.$$

Thus for most parameter values in the null, the asymptotic size of the critical set is correct. Finally consider $\theta \notin \Theta_r$, then the power property shows that

$$P_\theta(C_r) \to 1,$$

such that the test has asymptotic power 1.

We apply this to define the estimator $\hat{r}$, by

$$\{\hat{r} = 0\} = C_0^c,$$
$$\{\hat{r} = r\} = C_{r-1} \setminus C_r, \quad (r = 1,\dots,p-1)$$
$$\{\hat{r} = p\} = C_{p-1}.$$

This means that we test the hypotheses starting with Θ_0, and take r to be the subscript of the first non rejected hypothesis. It follows that for $\theta \in \Theta_r \setminus \Theta_{r-1}$,

$$P_\theta\{\hat{r} = i\} \to \begin{cases} 0 \;\;, & i = 0,1,\dots,r-1 \\ 1-\alpha, & i = r. \end{cases}.$$

Thus when the true parameter space is $\Theta_r \setminus \Theta_{r-1}$, then in the limit the estimate $\hat{r}$ takes on the correct value with probablility $1-\alpha$, and only takes larger values with positive probability.

IV. ILLUSTRATIVE EXAMPLES

In the first example we apply this set of ideas to make inference about the cointegration rank under the assumption that there is a linear trend in the data.

We consider the UK data analysed in Johansen and Juselius (1991). The data consists of quarterly observations from 1972.1 to 1987.2 of the five variables p_1 (a UK wholesale price index), p_2 (a trade weighted foreign wholesale price index), e_{12} (the UK effective exchange rate), i_1 (the three months treasury bill rate in UK), and i_2 (the three months Eurodollar interest rate). The detailed analysis in Johansen and Juselius (1991) involved fitting an autoregressive model to the logarithms with two lags, seasonal dummies and constant term. In addition current and lagged changes of the world oil price were included in the model as strictly exogenous variables. This model allows for a trend, as is reasonable from graphs of the data.

We identify Θ_r with H_r, and denote all the parameters in the model by ϑ. The test statistic for H_r in H_p, as given by (2.3), has the limit distribution (2.4) if in fact the cointegrating rank is r, that is if $H_r^0 = H_r \setminus H_{r-1}$ is the true model. If H_{r-1}^0 is true, and this is part of the null hypothesis H_r being tested, the limit distribution of (2.3) is different, and this should be taken into account when evaluating the size of the test as discussed in Section III.

The asymptotic distribution of T_r allowing for a linear trend is tabulated in Table A1 in Johansen and Juselius (1990). The test statistics for the UK data and the appropriate 95% quantiles are reported in Table 1.

TABLE 1

The results of a cointegration analysis of the UK data. The likelihood ratio test statistic T_r for the hypothesis H_r in H_p as well as the 95% quantiles $c_r(5\%)$ from table A1 in Johansen and Juselius (1990)

p–r	r	T_r	$c_r(5\%)$
5	0	80.75	68.91
4	1	49.42	47.18
3	2	29.26	29.51
2	3	11.67	15.20
1	4	5.19	3.96

The formal procedure is that one takes a level, 5%, say and starts at the top by rejecting H_0, since T_0 is larger than the corresponding quantile. In the next row we also reject H_1, since T_1 is larger than the corresponding quantile. Finally the third row contains a statistic which can not be rejected, which defines $\hat{r} = 2$. Thus the basic message is to start testing at the top.

Next we consider an example from Johansen and Juselius (1990) on money demand in Finland. The data consists of quarterly observations from 1958.1 to 1984.3, and contains four variables, m1 (money), y (income), i^m (the marginal rate of interest), and Δp (the inflation rate). In this example we illustrate how the procedure needs to be changed if a linear trend might be needed to describe the data. In this case we have to consider jointly the general hypothesis H_r of at most r cointegrating relations, and also the sub hypothesis H_r^* of at most r cointegrating relations but no linear trend, see (1.2). The hypotheses in question are partially ordered as shown on Table 2:

TABLE 2

The relation between the hypotheses H_r and H_r^* together with the test statistics for testing these against the unrestricted VAR model.

$$
\begin{array}{ccccc}
H_0^* & \subset & H_0 & & T_0^* & T_0 \\
 & & \cap & & & \\
H_1^* & \subset & H_1 & & T_1^* & T_1 \\
 & & \cap & & & \\
\vdots & & \vdots & & & \\
 & & \cap & & & \\
H_p^* & \subset & H_p & & T_p^* & T_p
\end{array}
$$

We determine a quantile $c_r(\alpha)$, such that

$$P_\theta\{T_r \geq c_r(\alpha)\} \to \alpha, \; \theta \in H_r^0 \setminus H_r^*,$$

when there are r cointegration relations and a linear trend. This quantile is determined from Table A1 in Johansen and Juselius (1990). Similarly we determine $c_r^*(\alpha)$ such that

$$P_\theta\{T_r^* \geq c_r^*(\alpha)\} \to \alpha,\ \theta \in H_r^* \cap H_r^0,$$

when there are r cointegration relations and no linear trend. This quantile is determined from Table A3 in Johansen and Juselius (1990). The critical regions are defined as

$$C_{2r} = \{T_0^* \geq c_0^*(\alpha),\ T_0 \geq c_0(\alpha),\ \ldots, T_r^* \geq c_r^*(\alpha),\ T_r \geq c_r(\alpha)\}$$
$$C_{2r-1} = \{T_0^* \geq c_0^*(\alpha),\ T_0 \geq c_0(\alpha),\ \ldots, T_r^* \geq c_r^*(\alpha)\}$$

The numbering of the critical regions corresponds to an ordering of the hypotheses from left to right and from top to bottom in Table 2. Then we test the hypotheses starting with H_0^* and stop the first time we do not reject. This defines the random variables $\hat{r}$ and L, which is 1 or 0, by the following definition

$$\{\hat{r} = r,\ L = 1\} = C_{2r-2} \cap \{T_r^* \geq c_r^*(\alpha),\ T_r < c_r(\alpha)\} = C_{2r-1} \setminus C_{2r},$$
$$\{\hat{r} = r,\ L = 0\} = C_{2r-2} \cap \{T_r^* < c_r^*(\alpha)\} = C_{2r-2} \setminus C_{2r-1}.$$

Thus $\hat{r}$ denotes the estimate of the cointegration rank, and L the presence of a linear trend. It then holds that for $\theta \in H_r^0 \setminus H_r^*$, that is if there are r cointegration relations and a linear trend, then

$$P_\theta\{\hat{r} = i, L = 1\} \leq P_\theta\{T_i < c_i(\alpha)\} \to 0, \quad (i = 0,1,\ldots,r-1)$$

since the test statistic T_i contains the p–i smallest eigenvalues and if $i < r$ then at least one of these is positive even in the limit, forcing T_i to ∞, such that the set $\{T_i < c_i(\alpha)\}$ tends to the empty set. We also find that

$$P_\theta\{\hat{r} = i, L = 1\} \to 1-\alpha, \qquad (i = r),$$

since T_i, T_i^*, $i = 0,1,\ldots,r-1$, as well as T_r^* all tend to ∞, and the set $\{T_r < c_r(\alpha)\}$ in the limit has probability $1-\alpha$.

Finally one finds that

$$P_\theta\{\hat{r} = r, L = 0\} = P_\theta\{C_{2r-1} \cap \{T_r^* < c_r^*(\alpha)\}\} \to 0,$$

since the presence of the linear trend forces T_r^* to ∞.

Thus it is seen that the procedure is consistent in the sense that a level α test will give the correct value of r and L in the limit with a probabilty of $1-\alpha$. Note that Table 2 shows a partial order of the relevant hypotheses, and the whole idea is simply to reject a hypothesis only if all hypotheses coming before are also rejeted.

TABLE 3

The results of a cointegration analysis of the Finnish data. The likelihood ratio test statistics T_r^* and T_r for the hypotheses H_r^* and H_r in H_p as well as the 95% quantiles $c_r^*(5\%)$ and $c_r(5\%)$ from table A3 and A1 in Johansen and Juselius (1990)

p–r	r	T_r^*	$c_r^*(5\%)$	T_r	$c_r(5\%)$
4	0	103.11	53.35	76.14	47.18
3	1	51.32	35.07	37.65	29.51
2	2	21.87	20.17	11.01	15.20
1	3	7.89	9.09	3.11	3.96

Reading from left to right and from top to bottom in Table 3 we find that H_0^* and H_0 have to be rejected at the 5% level, as well as H_1^* and H_1. The hypothesis H_2^* is also rejected, but H_2 is not. Thus we can conclude that $\hat{r} = 2$, and that there is a linear trend in the data (L = 1).

V. DISCUSSION

The purpose of this paper is to demonstrate how to use the tables in Johansen and Juselius (1990) for conducting inference about the cointegrating rank. The reason that inference is difficult is that the asymptotic distribution under the null of the test statistic depends on which parameter value one considers under the null. Thus the test is not similar. In the case of a cointegration analysis the limit distribution depends on the actual (true) number of cointegrating relations and also on the presence of a linear trend.

This problem is discussed in the statistical literature, see Pantula (1989) and Berger and Sinclair (1984). The solution proposed is to identify the sub hypotheses, which give different limit distributions, and construct a test statistic and a critical region for each of these sub hypotheses. The critical region for the test of the original null hypothesis is then the intersection of the critical regions constructed for each of the sub hypotheses or, in other words, the hypothesis in question is only rejected if all sub hypotheses are rejected.

This procedure is applied to some published examples of the determination of the cointegration rank.

One should point out that in practise the testing procedure is rarely so simple to formulate, but in order to discuss formal properties one has to formalize the procedure. There is of course nothing canonical about the 5% level usually applied, and the determination of the cointegration rank should also be based on the interpretation of the estimated cointegrating relations. Thus in the analysiFinnish data in Johansen and Juselius (1990) we chose $\hat{r} = 3$, since the third eigenvector had equal coefficients with opposite sign to m1 and y, and this allowed a very simple description of the data.

It is probably a very good idea to check the conclusions reached in the analysis of a given data set by trying out different values of r, to see in what sense the choice is critical for the conclusions. This procedure, however, is not so easy to formalize.

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