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Given Strength for the Poisson
and Binomial Distributions



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Summary

A collection of good two-, three- and seven-stage sampling plans for the Poisson distribution is given. For each plan 15 OC fractiles and the corresponding ASN values are tabulated. Efficiency in relation both to single and sequential sampling is discussed. An approximation formula for obtaining binomial OC fractiles from the corresponding Poisson fractiles is given.

Key words

Multiple sampling plans. Attribute sampling plans. Table of OC function. Table of ASN function. Poisson distribution. Binomial distribution. Curtailed sampling.

1. Notation.

Consider a Poisson process with mean occurrence rate per observational unit equal to λ . The probability of getting x events (defects) in n units is $g(x, n\lambda) = e^{-n\lambda} (n\lambda)^x / x!$ and the probability of getting c or less events is denoted by $G(c, n\lambda)$. The parameters (c, n) may take on non-negative real values if we interpret $G(c, n\lambda)$ as an incomplete gamma function. For $x < 0$ we set $g(x, n\lambda) = 0$.

A k -stage sampling plan, $k = 1, 2, \dots$, is defined by means of $3k$ integers (a_i, r_i, n_i) , $i = 1, \dots, k$, where $r_k = a_k + 1$, the three components denoting the acceptance numbers, the rejection numbers and the sample sizes, respectively. It is assumed that $a_i \leq a_{i+1}$, $r_i \leq r_{i+1}$ and $r_i - a_i \geq 2$ for $i = 1, \dots, k - 1$. The notation $a_i = -1$ is used to indicate that acceptance is not permitted in sample no. i . We shall use the first sample size n_1 as scale factor and therefore introduce the relative sample sizes $t_i = n_i / n_1$, $i = 1, \dots, k$. For short we shall write (b, n_1) for the whole set of $3k-1$ parameters.

The OC function will be denoted by $P_k(\lambda) = H_k(b, n_1 \lambda)$ and the ASN function by $\bar{n}_k(\lambda) = n_1 M_k(b, n_1 \lambda)$.

A sampling plan is said to be of strength $(\lambda_1, \alpha, \lambda_2, \beta)$ if it satisfies the equations $P_k(\lambda_1) = 1 - \alpha$ and $P_k(\lambda_2) = \beta$.

The single sampling plan and the sequential probability ratio test (SPRT), satisfying the same requirements are called OC equivalent to the given multiple plan. The parameters for the equivalent single sampling plan are denoted by (a_0, n_0) . The SPRT has the acceptance boundary $-a + \lambda_{12}n$ and the rejection boundary $r + \lambda_{12}n$, where $\lambda_{12} = (\lambda_2 - \lambda_1) / \{\ln(\lambda_2 / \lambda_1)\}$. For the SPRT the subscript s (sequential) will

be used analogous to k for the multiple sampling plan. The scale factor (analogous to n_1) is $n_s = 1 / \lambda_{12}$.

To compare the ASN functions of equivalent plans we introduce the relative efficiency

$$e_k(\lambda) = \bar{n}_s(\lambda) / \bar{n}_k(\lambda) = n_s M_s(a, r, n_s \lambda) / (n_1 M_k(b, n_1 \lambda))$$

All ASN functions considered are for fully curtailed inspection.

Percentage points for the OC function, also called OC fractiles, are denoted by the variable in question and a subscript indicating the percentage or fraction such as λ_p , $0 \leq p \leq 1$.

Solving the equation $H_k(b, n_1 \lambda_p^{(k)}) = P$ we get $n_1 \lambda_p^{(k)} = v_p(b)$, say. For the SPRT we get similarly $n_s \lambda_p^{(s)} = v_p(a, r)$, hoping that the two arguments will be sufficient to avoid confusion with $v_p(b)$. For single sampling, however, we need a special symbol and therefore introduce $n_0 \lambda_p^{(1)} = m_p(a_0)$.

We shall leave out the superscripts to λ_p when no confusion is possible.

We shall use the auxiliary variable $v = n_1 \lambda$ and consequently $n_i \lambda = t_i v$.

2. The OC and ASN functions.

For single sampling we have $P_1(\lambda) = G(a_0, n_0 \lambda)$ and for curtailed inspection

$$ASN/n_0 = \bar{n}_1(\lambda)/n_0 = G(a_0, n_0 \lambda) + \{(a_0+1)/(n_0 \lambda)\} \{1 - G(a_0+1, n_0 \lambda)\}.$$

The OC fractiles $m_p(a_0) = \frac{1}{2} \chi_{1-p}^2(2a_0+2)$ have been extensively tabulated, and a table of $\bar{n}_1(\lambda)/n_0$ has been given by Hald and Møller (1976 a).

New tables and approximations for the SPRT and references to previous papers have been given by Hald and Møller (1976 b).

For multiple sampling in the binomial case the OC and ASN functions have been derived by the Statistical Research Group, Columbia University (1948).

For the Poisson process the proof is completely analogous so we shall only state the results.

Let x_i denote the number of defects found in the i th sample, $x_{(i)} = x_1 + \dots + x_i$, $n_{(i)} = n_1 + \dots + n_i$ and $t_{(i)} = n_{(i)}/n_1$. Consider first the probability, $f_x^{(i+1)}(v)$ say, of getting a total of x defects in the first $i+1$ samples under the condition that the outcome of the previous samples has led to neither acceptance nor rejection, i.e.

$$f_x^{(i+1)} = \Pr \{x_{(i+1)} = x \mid a_j < x_{(j)} < r_j, j = 1, \dots, i\}.$$

This probability is easily found by the recursion formula

$$f_x^{(i+1)}(v) = \sum_{y=a_i+1}^{r_i-1} f_y^{(i)}(v) g(x-y, t_{i+1}v),$$

where $f_x^{(1)}(v) = g(x, v)$, $v = n_1 \lambda$. For completeness we set $a_0 = -1$, $r_0 = 1$ and define $f_x^{(0)} = 1$ for $x = 0$ and equal to 0 otherwise.

The probability of acceptance on the $(i+1)$ st sample is

$$P_a^{(i+1)}(\lambda) = \sum_{x=a_i+1}^{r_i-1} f_x^{(i)}(v) G(a_{i+1}-x, t_{i+1}v)$$

and the probability of rejection is

$$P_r^{(i+1)}(\lambda) = \sum_{x=a_i+1}^{r_i-1} f_x^{(i)}(v) \{1 - G(r_{i+1}-1-x, t_{i+1}v)\}$$

so that the probability that inspection stops at stage $i+1$ and not sooner equals $P_a^{(i+1)} + P_r^{(i+1)}$. The probability of acceptance, the OC function, becomes

$$P_k(\lambda) = \sum_{i=1}^k P_a^{(i)}(\lambda) = H_k(b, v).$$

Introducing the auxiliary function

$$\mu_x^{(i)}(v) = t_i G(a_i - x, t_i v) + \{(r_i - x)/v\} \{1 - G(r_i - x, t_i v)\},$$

which gives the average sample time divided by n_1 for inspection during stage i until a decision of acceptance or rejection is reached, we have

$$\bar{n}_k(\lambda)/n_1 = M_k(b, v) = \sum_{i=2}^k t_{(i-1)} \{P_a^{(i)} + P_r^{(i)}\} + \sum_{i=1}^k \sum_{x=a_{i-1}+1}^{r_{i-1}-1} f_x^{(i-1)}(v) \mu_x^{(i)}(v).$$

For $k = 1$ we have $\bar{n}_1(\lambda)/n_1 = \mu_0^{(1)}(v)$ in agreement with the result given above with the notation (a_0, n_0) .

3. Equivalence and efficiency.

Consider a multiple sampling plan (b, n_1) of strength $(\lambda_1, \alpha, \lambda_2, \beta)$ and the equivalent single and sequential sampling plans, (a_0, n_0) and (a, r, n_s) respectively. Expressing the equivalence in terms of OC fractiles we get three pairs of equations

$$n_1 \lambda_1 = v_{1-\alpha}(b) \quad \text{and} \quad n_1 \lambda_2 = v_\beta(b) \quad ,$$

$$n_0 \lambda_1 = m_{1-\alpha}(a_0) \quad \text{and} \quad n_0 \lambda_2 = m_\beta(a_0) \quad ,$$

$$n_s \lambda_1 = v_{1-\alpha}(a, r) \quad \text{and} \quad n_s \lambda_2 = v_\beta(a, r) \quad .$$

Setting $R = \lambda_2 / \lambda_1$ we have

$$v_\beta(b)/v_{1-\alpha}(b) = m_\beta(a_0)/m_{1-\alpha}(a_0) = v_\beta(a, r)/v_{1-\alpha}(a, r) = R$$

so that for given (R, α, β) or (b, α, β) we may solve the equations and find a_0 and (a, r) . Furthermore,

$$n_0/n_1 = m_\beta(a_0)/v_\beta(b)$$

and

$$n_s/n_1 = v_\beta(a, r)/v_\beta(b)$$

which means that the ratios of the scale factors (sample sizes) are functions of (b, α, β) .

To find the efficiency of a multiple sampling plan in relation to the equivalent SPRT for $\lambda = \lambda_P = v_P(b)/n_1$, the P fractile for the multiple plan, we note that $n_1 \lambda_P$ and $n_s \lambda_P = (n_s/n_1)(n_1 \lambda_P)$ both are functions of P and b only so that the efficiency becomes a function of P and b .

It is well known that among equivalent tests the SPRT is the most efficient in the sense that it minimizes the ASN function for the two given values of the parameter. For other values of the parameter it is a somewhat doubtful procedure to use $e_k(\lambda)$ as measure of efficiency, in particular because the OC functions take on different values. In lack of something better we shall, however, use $e_k(\lambda)$ for comparing the two ASN functions.

4. Construction of multiple sampling plans.

A multiple sampling plan of given strength having high ASN-efficiency is usually constructed from the corresponding SPRT by truncation and grouping, using successive samples of the same size and adjusting the acceptance and rejection numbers by trial and error to obtain the strength required. Hill (1973) has explained how the seven-stage plans in Military Standard 105D have been constructed in this way by fitting a sequential plan to the given single sampling plan, using $n_1 = \dots = n_7 = n_0/4$ and modifying the last two pairs of acceptance and rejection numbers to obtain a wedge-shaped continuation region. Military Standard does not give the exact OC functions for the multiple plans but contain graphs of the ASN/n_0 .

Enters and Hamaker (1951) have discussed plans with $n_1 = \dots = n_k$, boundaries of the form $a_i = -\hat{a} + si$, $r_i = \hat{r} + si$, $r_i \leq r_k$, for $i = 1, \dots, k-1$, and $r_k = sk$, so that the number of parameters is reduced from $3k-1$ to 4, namely $(\hat{a}, \hat{r}, s, n_1)$. They conclude that the greater part of the saving by sequential sampling is obtained by means of multiple sampling for $k = 8$, and they give a collection of 13 eighth-stage plans of this type supplemented by $n_1^{\lambda.95}$, $n_1^{\lambda.05}$ and efficiencies in relation to single sampling for $P = 1, \frac{1}{2}$ and 0.

We have constructed plans for $k = 2, 3$ and 7 having high efficiency in relation to the equivalent SPRT for $\alpha = 0.05$ and $\beta = 0.10$. In all cases we have used $n_1 = \dots = n_k$. A good multiple sampling plan is characterized by having small values of $\bar{n}_k(\lambda_{.95})/\bar{n}_s(\lambda_{.95})$ and $\bar{n}_k(\lambda_{.10})/\bar{n}_s(\lambda_{.10})$, and to simplify matters further we have used the average of these quantities, i_k say, as criterion. Hence, for each collection of multiple sampling plans our main objective has been to select plans with the smallest possible i_k .

Consider first the problem of finding a collection of good double sampling plans. All double sampling plans may be ordered according to increasing values of a_2 and, for each a_2 , ordered according to increasing values of a_1 and r_1 . For each $b = (a_1, r_1, a_2)$ we find the equivalent $(a_0, n_1/n_0)$ and $(a, r, n_1/n_s)$ as described in Section 3. Each double sampling plan may then be characterized by a point (a_0, i_2) and extending this point set by randomization it will be seen that the "optimum plans" correspond to the lower boundary of the convex set. To find the non-randomized optimum plans we rearrange the plans according to increasing values of a_0 (or decreasing values of $R = \lambda_{.10}/\lambda_{.95}$) for all $a_0 < 30$, say, and select the plans on the convex lower boundary of the tabulated values of (a_0, i_2) . This procedure leads to a collection of about 30 non-randomized optimum plans with $a_0 < 30$, but unfortunately this collection is unsatisfactory in two respects. Firstly, the spacing of the a_0 values is very irregular and, secondly, the decision numbers (a_1, r_1, a_2) are not increasing with a_0 . It seems reasonable to require that we should be able to meet the strength specifications at least as well by means of a set of double sampling plans as by single sampling which means that the differences between successive values of a_0 should be at most 1. On the other hand we are not interested in having several plans with nearly the same values of a_0 , i.e. nearly the same OC

functions, and we therefore require that the differences between successive values of a_0 , should be at least 0.2. Furthermore we impose the condition that the decision numbers should be increasing functions of a_0 . To achieve these goals we have removed some of the optimum plans and added some nearly optimum ones to obtain a collection with about 30 plans for $a_0 < 20$ and about 40 plans for $a_0 < 30$. The plans for $k = 3$ have been constructed by the same method.

For $k = 7$, however, an enumeration of all possible plans with $a_0 < 30$ would lead to a very large number of plans and a correspondingly large amount of computation. We have therefore limited the search for good plans by imposing some natural restrictions on the initial collection of plans. Considering a SPRT truncated at $n = kn_1$ it is natural to accept if the number of defects at the point of truncation is at most $\lambda_{12}kn_1$. We therefore put

$$a_k + \frac{1}{2} = \lambda_{12}kn_1 = \{(R-1)/(\ln R)\} m_{1-\alpha} (a_0)kn_1/n_0$$

and as a first approximation

$$a_i \approx -a + i\lambda_{12}n_1 = -a + (a_k + \frac{1}{2})(i/k) ,$$

a similar formula being valid for r_i with the modification that $r_i \leq a_k + 1$. Setting $7n_1 = 1.75 n_0$ as in Military Standard 105D the first equation may be solved with respect to a_0 for any given value of a_7 , since R is a function of a_0 , and from a_0 we find the equivalent SPRT, i.e. (a, r) . The problem is then reduced to derive integer values of a_i and r_i from the approximations above. It turns out that rounding to the nearest integer usually gives good results but this is not always so and the resulting value of a_0 may be rather different from the starting value. After some experimentation we have found that the following method of "compressed limits" in general gives good results: The parameters (a, r) should be replaced by $(a-\delta, r-\delta)$, $\delta > 0$, and (a_i, r_i) should then be found from the corresponding approximation formulas by

rounding down from the lower and up from the upper boundary. The constant δ should be determined such that the final multiple sampling plan has the same (or a slightly larger) value of a_0 as the starting value. Numerical investigations have shown that this criterion leads to plans of high efficiency. We have also modified the last pair of decision numbers to obtain a wedge-shaped continuation region if this gives higher efficiency. Setting $a_k = 2.0, 2.5 (0.5) 50.0$ and determining for each a_k a multiple plan according to the method indicated above we get a collection of plans from which we select the nearly optimum ones using the same principles as for $k = 2$ and 3 . Notice that the factor 1.75 is used only as a starting value and that for each set of decision numbers chosen we find the corresponding value of n_1/n_0 .

The three collections of nearly optimum multiple plans are given at the end of the paper. For each plan 15 OC fractiles, $v_p = n_1 \lambda_p$, and corresponding values of $M(b, n_1 \lambda_p) = \bar{n}(\lambda_p)/n_1$ have been tabulated. The tables also contain $R = \lambda_{.10} / \lambda_{.95}$ and $(a_0, n_1/n_0)$ for the equivalent single sampling plan with $\alpha = 0.05$ and $\beta = 0.10$. It is therefore easy to find the efficiency relative to single sampling since $ASN/n_0 = M n_1/n_0$.

To demonstrate the efficiency of the multiple sampling plans we have computed the values of $\bar{n}_k(\lambda_p) / \bar{n}_s(\lambda_p)$ for $P = 0.95, 0.50$ and 0.10 for each plan. Apart from small irregularities due to the discreteness of the decision numbers this ratio is a slowly increasing function of a_0 as shown in Table 1 where we have smoothed out the irregularities by taking averages for the three plans closest to the values of a_0 indicated.

1. Table of $\bar{n}_k(\lambda_p) / \bar{n}_s(\lambda_p)$.

a_0	P = 0.95			P = 0.50			P = 0.10		
	k = 2	3	7	2	3	7	2	3	7
5	1.29	1.15	1.09	1.05	1.00	0.99	1.22	1.14	1.05
10	1.32	1.18	1.08	1.05	1.01	0.99	1.21	1.17	1.05
15	1.36	1.22	1.09	1.06	1.01	0.99	1.24	1.17	1.07
20	1.37	1.23	1.09	1.07	1.03	1.00	1.27	1.19	1.09
25	1.39	1.24	1.10	1.08	1.03	1.01	1.29	1.19	1.09

At the two specified points and for $5 \leq a_0 \leq 25$ the increase in the ASN as compared to sequential sampling is roughly 10% for seven-stage plans, 20% for three-stage plans and 30% for two-stage plans. The results for $P = 0.50$ show that the ASN curves for the multiple sampling plans are more flat than for the sequential plans.

The efficiency relative to single sampling has been shown in Table 2.

2. Table of $100 \bar{n}_k(\lambda_p) / n_0$.

a_0	P = 0.95			P = 0.50			P = 0.10		
	k = 2	3	7	2	3	7	2	3	7
5	76	69	65	79	77	75	58	54	50
10	74	66	60	80	77	75	60	58	52
15	73	66	58	81	77	76	62	59	54
20	72	65	58	81	78	77	65	60	55
25	72	65	57	82	79	78	66	61	56

From the tables at the end of the paper it will be seen that n_1/n_0 is nearly constant for $a_0 > 5$. The average values for $5 < a_0 < 30$ are 0.562, 0.423 and 0.243 for $k = 2, 3$ and 7 , respectively, so that

the maximum sample sizes, kn_1 , become 1.12, 1.27 and 1.70 times the equivalent single sample size.

5. Asymptotic results and approximations for double sampling plans.

For double sampling the present paper may be considered as a continuation of the paper by Hald (1975) on optimum double sampling plans for the normal distribution. However, in the present paper we have set $n_1 = n_2$ and chosen the remaining parameters in an optimum way under this restriction whereas in the previous paper we used the optimum value of $\rho = n_1 / (n_1 + n_2)$. We shall now supplement the results given in Table 3 of the previous paper by some new results to demonstrate that restricting ρ to the value $\frac{1}{2}$ does not essentially decrease the efficiency of the plans if only the other parameters are chosen optimally.

Table 3 shows $\bar{n}(\theta_1)/n_0$ for some double sampling plans of strength $(\theta_1, \alpha, \theta_2, \alpha)$ minimizing $\bar{n}(\theta_1)$ and also $\bar{n}(\theta_2)$ because of the symmetry of the problem. The last section of the table shows the optimum plans which are identical to the results in Table 3 of the previous paper. The first and the second section of the table show the corresponding results for a fixed value of ρ , in the first section $\rho = 0.436$, which is the optimum value for $\alpha = 0.05$, and in the second section $\rho = \frac{1}{2}$. It will be seen that the increase in $\bar{n}(\theta_1)/n_0$ is small. For $\rho = 0.436$ the increase is less than 1%. For $\rho = \frac{1}{2}$ the largest increase is 4% which occurs for $\alpha = 0.001$, for the ordinarily used values of α the increase is less than 2%. Similar results hold for $\alpha \neq \beta$ and also for the tests minimizing $\max_{\theta} \bar{n}(\theta)$. Hence, the loss in efficiency by using $\rho = \frac{1}{2}$ as in the present paper is small.

3. Double sampling tests of strength $(\theta_1, \alpha, \theta_2, \alpha)$ minimizing $\bar{n}(\theta_1)$.

α	$\rho = .436$			$\rho = .5$			Optimal ρ			
	y	n/n ₀	$\bar{n}(\theta_1)/n_0$	y	n/n ₀	$\bar{n}(\theta_1)/n_0$	ρ	y	n/n ₀	$\bar{n}(\theta_1)/n_0$
.001	1.124	1.112	.580	1.106	1.065	.599	.395	1.133	1.159	.576
.005	.970	1.138	.623	.958	1.087	.636	.408	.975	1.170	.621
.010	.903	1.151	.647	.893	1.098	.658	.415	.907	1.174	.646
.025	.815	1.170	.686	.805	1.115	.693	.426	.817	1.180	.685
.050	.749	1.185	.721	.738	1.130	.726	.436	.749	1.186	.721
.100	.684	1.202	.762	.672	1.148	.765	.447	.681	1.193	.762
.200	.621	1.222	.804	.607	1.168	.806	.457	.616	1.203	.804
.500	.567	1.242	.842	.551	1.188	.842	.465	.560	1.216	.842

For $\alpha = 0.05$, $\beta = 0.10$ and $\rho = \frac{1}{2}$ we find $n_1/n_0 = 0.566$, $y_a = 0.599$ and $y_r = 0.859$. Using the asymptotic theory sketched in Section 6 of the previous paper we get that the expected number of defects in the first sample equals $d = 0.566 (a_0+1)$ and that $a_1 \approx d - 1.1 - 0.599 \sqrt{d}$, $r_1 \approx d + 0.859 \sqrt{d}$ and $a_2 \approx 2d - 1.5$, where the continuity corrections 1.1 and 1.5 (instead of 1) have been determined numerically from the table of double sampling plans to get a good fit. Rounding should be to the nearest integer. Applying these formulas to the 42 tabulated plans it will be seen that the formula for r_1 gives the correct result in all cases whereas the two other formulas give the correct result in 35 and 33 cases, respectively, the error being + 1 or - 1 in the remaining cases. We may therefore use these formulas to obtain double sampling plans of high efficiency for $a_0 > 30$ and also for smaller values of a_0 if more plans are needed than those given in the table.

6. Use of the tables.

A collection of multiple sampling plans ordered according to values of $R(b, \alpha, \beta) = v_\beta(b)/v_{1-\alpha}(b)$, so that $R(b_1, \alpha, \beta) > R(b_2, \alpha, \beta) > \dots$, may be used to determine the plan of strength $(\lambda_1, \alpha, \lambda_2; \beta)$ or the one immediately stronger in the same way as a similar table of single sampling plans. The inequality $R(b_{i-1}, \alpha, \beta) > \lambda_2/\lambda_1 \geq R(b_i, \alpha, \beta)$ gives the decision numbers b_i and the first sample size satisfies the inequality $v_\beta(b_i)/\lambda_2 \leq n_1 \leq v_{1-\alpha}(b_i)/\lambda_1$. Choosing n_1 equal to the lower limit the plan will have $P(\lambda_2) = \beta$ and $1-P(\lambda_1) \leq \alpha$ and choosing n_1 equal to the upper limit we get $1-P(\lambda_1) = \alpha$ and $P(\lambda_2) \leq \beta$. Only for $\lambda_2/\lambda_1 = R(b_i, \alpha, \beta)$ the solution is unique.

As an example we consider the problem of testing $\lambda_1 = 0.01$ against the alternative $\lambda_2 = 0.03$ for $\alpha = 0.05$ and $\beta = 0.10$ choosing n_1 so that

the producer's risk becomes exactly 0.05 and the consumer's risk therefore at most 0.10. From the table for $k = 7$ we see that the solution is obtained for $R(b, \alpha, \beta) = 2.922$ with a corresponding $a_0 = 7.17$. From $v_{.95} = n_1 \lambda_{.95} = 0.998$ we get $n_1 = 0.998 / 0.01 = 99.8$ and the corresponding ASN becomes $99.8 \times 2.53 = 252$. Proceeding in this manner we get the results shown in Table 4.

4. Table of ASN values for sampling plans having $P(0.01) = 0.95$ and $P(0.03)$ less than but as near as possible to 0.10.

ASN($\lambda_p^{(k)}$) for

k	R	a_0	P = 0.95	P = 0.50	P = 0.10
1	2.96	7.00	398	398	398
1*	2.96	7.00	395	349	264
2	2.95	7.01	302	314	226
3	2.97	6.92	269	300	209
7	2.92	7.17	252	316	209

1* denotes curtailed single sampling.

It should be noted that the ASN values are not directly comparable apart from the case $P = 0.95$ where $\lambda_{.95} = 0.01$ for all values of k . In the other cases the values of λ_p varies slightly because the OC curves have slightly varying shapes depending on k and because the plans are not of exactly the same strength. We have that $\lambda_{.10} = 0.01 R$ and other values of λ_p may be found from v_p in the appropriate tables.

For the SPRT it has been demonstrated by Hald and Møller (1976b) that the table for $\alpha = 0.05$ and $\beta = 0.10$ may be considered as a Master Table in the sense that the plans tabulated may be used also for other values of α and β without an essential loss in efficiency. This is true also for the tables of multiple sampling plans. Hence, by computing $R(b, \alpha, \beta)$

from the tabulated fractiles for $\alpha \neq 0.05$ and/or $\beta \neq 0.10$ we may use the same table to find plans of strength $(\lambda_1, \alpha, \lambda_2, \beta)$ for the usually employed values of (α, β) .

7. Multiple sampling plans for the binomial distribution.

As shown by Hald (1975) the Poisson tables may be used to find the solution to the corresponding binomial problem with sufficient accuracy for most practical problems in sampling inspection. To find a plan of strength $(p_1, \alpha, p_2, \beta)$ the tables should be entered with the argument $R = (p_2/p_1)\{1 + \frac{1}{2}(p_2 - p_1)\}$ instead of p_2/p_1 and from the fractiles of the corresponding plan the limits for n_1 may be found as

$$(v_\beta/p_2) - \frac{1}{2}(v_\beta - f) \leq n_1 \leq (v_{1-\alpha}/p_1) + \frac{1}{2}(f - v_{1-\alpha}),$$

where $f = a_0 n_1 / n_0$ may be found in the table for $\alpha = 0.05$ and $\beta = 0.10$. Since f does not change much with (α, β) the tabulated values will give sufficient accuracy.

In general, the binomial OC fractiles, p_p say, may be found with good approximation from the Poisson fractiles for the plan with the same decision numbers as

$$p_p = v_p / \{n_1 + \frac{1}{2}(v_p - f)\}.$$

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Table 5.

Double sampling plans with $n_1 = n_2$.

Poisson OC fractiles, $v = n_1 \lambda$, and ASN = $n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	Probability of Acceptance													v	M		
				.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010			.005	.001
12.05 .878	.672	0	1 2	.026	.060	.086	.141	.207	.310	.484	1.01	1.86	2.49	3.12	3.77	4.65	5.32	6.91	v	M
8.856 1.32	.642	0	2 2	.044	.099	.140	.222	.316	.457	.680	1.29	2.19	2.80	3.40	4.00	4.81	5.44	6.96	v	M
6.793 1.89	.565	0	2 3	.100	.177	.228	.325	.429	.580	.810	1.42	2.31	2.92	3.50	4.09	4.87	5.49	6.98	v	M
5.357 2.63	.551	0	3 3	.171	.289	.363	.496	.635	.827	1.11	1.82	2.78	3.40	3.99	4.55	5.29	5.85	7.20	v	M
4.976 2.92	.526	0	3 4	.218	.342	.419	.555	.696	.889	1.17	1.88	2.84	3.46	4.04	4.61	5.34	5.90	7.23	v	M
4.397 3.52	.597	1	4 4	.354	.530	.635	.817	1.00	1.25	1.60	2.46	3.63	4.40	5.13	5.85	6.81	7.54	9.27	v	M
4.012 4.07	.590	1	5 4	.414	.630	.758	.975	1.19	1.47	1.87	2.80	4.01	4.77	5.48	6.17	7.06	7.75	9.37	v	M
3.678 4.70	.546	1	5 5	.549	.769	.896	1.11	1.32	1.60	1.99	2.91	4.11	4.86	5.56	6.24	7.12	7.79	9.39	v	M
3.377 5.46	.541	1	6 5	.673	.939	1.09	1.34	1.58	1.89	2.32	3.32	4.57	5.33	6.04	6.70	7.55	8.18	9.66	v	M
3.258 5.82	.523	1	6 6	.760	1.02	1.17	1.42	1.65	1.96	2.39	3.38	4.62	5.39	6.09	6.75	7.59	8.22	9.69	v	M
3.106 6.37	.564	2	7 6	.949	1.26	1.44	1.72	1.99	2.35	2.83	3.94	5.33	6.20	7.00	7.77	8.78	9.54	11.3	v	M
2.954 7.01	.561	2	8 6	1.05	1.42	1.61	1.93	2.24	2.62	3.14	4.31	5.74	6.61	7.40	8.15	9.11	9.83	11.5	v	M
2.836 7.61	.534	2	8 7	1.21	1.56	1.75	2.06	2.36	2.73	3.24	4.40	5.82	6.68	7.47	8.21	9.16	9.87	11.5	v	M
2.778 7.94	.580	3	9 7	1.39	1.79	2.01	2.36	2.70	3.12	3.68	4.95	6.52	7.49	8.38	9.24	10.4	11.2	13.1	v	M
2.690 8.50	.578	3	10 7	1.48	1.93	2.17	2.56	2.92	3.38	3.98	5.30	6.90	7.86	8.74	9.56	10.6	11.4	13.3	v	M
2.562 9.44	.594	4	11 8	1.86	2.35	2.62	3.03	3.42	3.90	4.54	5.97	7.70	8.76	9.73	10.7	11.9	12.8	14.9	v	M
2.505 9.93	.594	4	12 8	1.93	2.47	2.76	3.21	3.63	4.15	4.82	6.30	8.05	9.10	10.0	10.9	12.1	13.0	14.9	v	M
2.400 11.0	.555	4	12 9	2.22	2.73	3.00	3.43	3.83	4.33	4.98	6.43	8.15	9.19	10.1	11.0	12.1	13.0	15.0	v	M
2.343 11.6	.554	4	13 9	2.35	2.91	3.21	3.67	4.10	4.63	5.32	6.81	8.57	9.60	10.5	11.4	12.5	13.3	15.2	v	M
2.287 12.3	.532	4	13 10	2.55	3.08	3.37	3.81	4.23	4.74	5.42	6.90	8.64	9.67	10.6	11.5	12.5	13.3	15.2	v	M
2.271 12.6	.565	5	14 10	2.76	3.35	3.66	4.14	4.59	5.14	5.86	7.44	9.30	10.4	11.4	12.4	13.6	14.5	16.6	v	M
2.229 13.2	.564	5	15 10	2.87	3.52	3.85	4.37	4.85	5.43	6.19	7.81	9.70	10.8	11.8	12.7	13.9	14.8	16.8	v	M

Table 5.

Double sampling plans with $n_1 = n_2$.

Poisson OC fractiles, $v = n_1 \lambda$, and $ASN = n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	Probability of Acceptance																v	M
				.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001			
2.171 14.1	.574	6 11	16 17	3.31 1.05	3.98 1.11	4.33 1.14	4.86 1.20	5.36 1.26	5.96 1.33	6.75 1.39	8.45 1.40	10.4 1.22	11.6 1.08	12.7 .966	13.7 .872	15.0 .774	16.0 .715	18.2 .614	v	M	
2.140 14.7	.574	6 11	17 18	3.41 1.06	4.14 1.12	4.51 1.16	5.08 1.23	5.61 1.30	6.24 1.36	7.06 1.42	8.81 1.40	10.8 1.20	12.0 1.06	13.1 .947	14.0 .856	15.3 .763	16.2 .707	18.3 .610	v	M	
2.086 15.7	.546	6 12	17 18	3.71 1.08	4.39 1.15	4.74 1.20	5.29 1.27	5.79 1.34	6.40 1.41	7.20 1.48	8.92 1.49	10.9 1.30	12.1 1.16	13.1 1.04	14.1 .945	15.3 .842	16.2 .779	18.3 .669	v	M	
2.055 16.4	.563	7 13	18 19	4.11 1.06	4.81 1.11	5.18 1.15	5.75 1.21	6.27 1.27	6.91 1.35	7.74 1.42	9.54 1.45	11.6 1.30	12.9 1.17	14.0 1.05	15.1 .951	16.4 .846	17.4 .782	19.7 .671	v	M	
2.015 17.3	.553	7 13	19 20	4.31 1.07	5.06 1.14	5.44 1.18	6.04 1.25	6.58 1.31	7.24 1.39	8.10 1.46	9.93 1.47	12.0 1.29	13.3 1.15	14.4 1.03	15.4 .938	16.7 .836	17.7 .775	19.9 .669	v	M	
1.989 18.0	.569	8 14	20 21	4.74 1.05	5.50 1.10	5.90 1.14	6.52 1.20	7.08 1.26	7.76 1.33	8.65 1.40	10.6 1.44	12.8 1.29	14.1 1.16	15.3 1.04	16.4 .944	17.8 .841	18.8 .778	21.3 .671	v	M	
1.957 18.9	.559	8 14	21 22	4.92 1.06	5.74 1.12	6.16 1.16	6.80 1.23	7.38 1.30	8.09 1.37	9.00 1.44	10.9 1.45	13.2 1.28	14.4 1.14	15.6 1.03	16.7 .931	18.0 .832	19.1 .772	21.4 .669	v	M	
1.934 19.6	.574	9 15	22 23	5.37 1.05	6.21 1.10	6.63 1.13	7.29 1.19	7.89 1.25	8.62 1.32	9.57 1.39	11.6 1.42	13.9 1.27	15.3 1.15	16.5 1.03	17.7 .937	19.2 .836	20.2 .775	22.7 .671	v	M	
1.907 20.5	.566	9 15	23 24	5.55 1.06	6.43 1.11	6.88 1.15	7.57 1.22	8.19 1.28	8.94 1.35	9.91 1.42	11.9 1.43	14.3 1.27	15.6 1.13	16.8 1.02	18.0 .926	19.4 .829	20.4 .770	22.8 .669	v	M	
1.888 21.2	.580	10 16	24 25	6.02 1.04	6.92 1.09	7.37 1.12	8.08 1.18	8.72 1.24	9.49 1.30	10.5 1.38	12.6 1.41	15.0 1.26	16.5 1.14	17.7 1.03	19.0 .931	20.5 .832	21.6 .773	24.2 .672	v	M	
1.864 22.0	.572	10 16	25 26	6.18 1.05	7.13 1.10	7.61 1.14	8.35 1.20	9.00 1.27	9.80 1.34	10.8 1.40	12.9 1.42	15.4 1.25	16.8 1.12	18.0 1.01	19.2 .922	20.7 .826	21.8 .769	24.3 .670	v	M	
1.842 22.9	.555	10 17	25 26	6.43 1.06	7.33 1.12	7.79 1.16	8.50 1.23	9.14 1.29	9.92 1.37	10.9 1.44	13.0 1.48	15.4 1.34	16.8 1.21	18.1 1.09	19.3 .995	20.7 .891	21.8 .827	24.3 .717	v	M	
1.820 23.9	.548	10 17	26 27	6.64 1.07	7.59 1.14	8.07 1.19	8.81 1.26	9.47 1.33	10.3 1.40	11.3 1.48	13.4 1.49	15.8 1.33	17.2 1.19	18.5 1.08	19.6 .980	21.0 .880	22.1 .819	24.5 .714	v	M	
1.805 24.5	.559	11 18	27 28	7.10 1.06	8.06 1.11	8.55 1.15	9.30 1.22	9.98 1.28	10.8 1.35	11.8 1.43	14.0 1.47	16.5 1.33	18.0 1.20	19.3 1.08	20.5 .988	22.0 .886	23.2 .824	25.7 .716	v	M	
1.786 25.5	.553	11 18	28 29	7.30 1.07	8.31 1.13	8.82 1.17	9.60 1.25	10.3 1.31	11.1 1.39	12.2 1.46	14.4 1.48	16.9 1.32	18.4 1.18	19.7 1.07	20.8 .975	22.3 .876	23.4 .816	25.9 .713	v	M	
1.772 26.2	.563	12 19	29 30	7.78 1.05	8.80 1.11	9.32 1.14	10.1 1.21	10.8 1.27	11.7 1.34	12.8 1.42	15.1 1.46	17.7 1.32	19.2 1.19	20.5 1.08	21.8 .982	23.4 .881	24.5 .820	27.2 .716	v	M	
1.755 27.1	.557	12 19	30 31	7.97 1.06	9.04 1.12	9.58 1.16	10.4 1.23	11.1 1.30	12.0 1.37	13.1 1.45	15.4 1.47	18.0 1.31	19.5 1.18	20.9 1.06	22.1 .970	23.6 .873	24.7 .814	27.3 .713	v	M	
1.743 27.8	.568	13 20	31 32	8.47 1.05	9.54 1.10	10.1 1.14	10.9 1.20	11.7 1.26	12.5 1.33	13.7 1.41	16.1 1.45	18.8 1.31	20.3 1.18	21.7 1.07	23.0 .977	24.6 .878	25.8 .818	28.6 .715	v	M	
1.729 28.6	.562	13 20	32 33	8.65 1.06	9.78 1.12	10.3 1.16	11.2 1.22	12.0 1.29	12.9 1.36	14.0 1.43	16.4 1.46	19.1 1.30	20.7 1.17	22.0 1.06	23.3 .966	24.9 .870	26.0 .812	28.7 .713	v	M	
1.717 29.3	.572	14 21	33 34	9.16 1.05	10.3 1.10	10.9 1.13	11.7 1.19	12.5 1.25	13.4 1.32	14.6 1.40	17.1 1.44	19.9 1.30	21.5 1.18	22.9 1.06	24.2 .972	25.9 .874	27.2 .816	30.0 .715	v	M	

Table 6.

Three-stage sampling plans with $n_1 = n_2 = n_3$.

Poisson OC fractiles, $v = n_1 \lambda$, and $ASN = n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	Probability of Acceptance															
					.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
27.13 .223	.486	-1 1	0 2	1 2	.001 2.00	.005 2.00	.010 1.99	.024 1.99	.048 1.97	.094 1.94	.182 1.86	.476 1.54	.961 1.08	1.30 .843	1.64 .676	1.97 .557	2.40 .447	2.73 .387	3.50 .293	v M
11.22 .967	.368	-1 2	0 2	1 2	.016 2.03	.037 2.07	.053 2.09	.086 2.13	.126 2.17	.189 2.20	.293 2.20	.599 2.00	1.08 1.56	1.41 1.30	1.74 1.11	2.06 .953	2.48 .801	2.80 .712	3.55 .563	v M
9.041 1.29	.584	0 2	1 3	2 3	.043 1.04	.093 1.09	.129 1.12	.199 1.18	.279 1.24	.396 1.30	.581 1.35	1.10 1.28	1.92 .987	2.52 .796	3.14 .649	3.78 .539	4.65 .435	5.32 .379	6.91 .290	v M
7.935 1.53	.561	0 2	1 4	3 4	.045 1.04	.102 1.10	.146 1.14	.232 1.21	.330 1.28	.471 1.36	.685 1.43	1.24 1.36	2.05 1.04	2.62 .832	3.20 .674	3.81 .555	4.66 .443	5.33 .383	6.92 .291	v M
6.506 2.00	.494	0 3	1 3	3 4	.098 1.10	.172 1.19	.220 1.24	.309 1.33	.404 1.41	.539 1.51	.741 1.59	1.28 1.55	2.07 1.26	2.63 1.06	3.21 .900	3.82 .770	4.66 .638	5.33 .560	6.92 .433	v M
5.587 2.47	.449	0 3	1 4	3 4	.140 1.15	.227 1.26	.282 1.32	.379 1.43	.480 1.52	.620 1.63	.826 1.73	1.36 1.73	2.13 1.41	2.68 1.18	3.24 .985	3.84 .826	4.67 .667	5.33 .577	6.92 .438	v M
4.806 3.08	.451	0 3	2 5	4 5	.181 1.17	.304 1.27	.378 1.33	.507 1.43	.637 1.52	.811 1.61	1.06 1.69	1.67 1.66	2.51 1.35	3.06 1.13	3.61 .945	4.15 .801	4.91 .656	5.51 .573	6.98 .438	v M
4.525 3.37	.451	0 3	2 5	5 6	.188 1.18	.327 1.29	.413 1.36	.564 1.48	.715 1.57	.913 1.67	1.19 1.75	1.84 1.67	2.69 1.32	3.24 1.09	3.76 .918	4.28 .783	4.99 .647	5.56 .568	6.99 .437	v M
4.043 4.02	.443	0 4	3 5	5 6	.330 1.28	.484 1.39	.574 1.45	.727 1.54	.879 1.62	1.08 1.70	1.36 1.76	2.05 1.71	2.96 1.41	3.55 1.21	4.11 1.05	4.65 .916	5.36 .782	5.91 .701	7.24 .562	v M
3.633 4.80	.483	1 4	3 7	7 8	.424 1.07	.650 1.16	.781 1.22	.995 1.32	1.20 1.42	1.46 1.53	1.81 1.63	2.62 1.61	3.67 1.30	4.36 1.08	5.04 .907	5.75 .769	6.71 .632	7.47 .556	9.24 .438	v M
3.321 5.62	.453	1 5	4 7	7 8	.625 1.14	.849 1.23	.973 1.28	1.18 1.38	1.37 1.47	1.62 1.57	1.97 1.67	2.79 1.70	3.85 1.45	4.55 1.25	5.23 1.07	5.92 .929	6.84 .780	7.56 .692	9.27 .549	v M
3.169 6.13	.438	1 5	4 7	8 9	.662 1.15	.909 1.25	1.05 1.32	1.27 1.42	1.48 1.52	1.75 1.63	2.11 1.72	2.95 1.72	4.01 1.43	4.69 1.23	5.34 1.06	5.99 .919	6.88 .776	7.58 .690	9.28 .548	v M
2.974 6.92	.422	1 5	4 8	9 10	.720 1.17	1.01 1.30	1.17 1.38	1.42 1.51	1.66 1.62	1.95 1.74	2.34 1.84	3.20 1.81	4.26 1.48	4.92 1.26	5.54 1.08	6.15 .937	6.99 .788	7.65 .700	9.29 .553	v M
2.808 7.77	.415	1 6	5 8	10 11	.928 1.25	1.22 1.37	1.38 1.44	1.64 1.56	1.88 1.66	2.18 1.77	2.59 1.86	3.50 1.84	4.60 1.54	5.27 1.34	5.90 1.17	6.50 1.04	7.29 .895	7.91 .809	9.43 .656	v M
2.683 8.54	.392	1 6	5 9	10 11	1.02 1.28	1.33 1.42	1.49 1.50	1.75 1.62	1.99 1.73	2.30 1.85	2.70 1.95	3.60 1.94	4.68 1.64	5.35 1.42	5.96 1.23	6.55 1.08	7.33 .926	7.94 .831	9.44 .665	v M
2.608 9.08	.426	2 7	6 10	11 12	1.29 1.15	1.62 1.26	1.80 1.32	2.08 1.42	2.34 1.53	2.66 1.64	3.10 1.77	4.09 1.83	5.31 1.59	6.09 1.39	6.84 1.21	7.59 1.06	8.60 .895	9.39 .798	11.3 .639	v M
2.510 9.89	.423	2 7	7 11	12 13	1.41 1.18	1.79 1.29	1.98 1.35	2.29 1.46	2.57 1.55	2.93 1.67	3.39 1.78	4.42 1.82	5.68 1.57	6.46 1.37	7.19 1.19	7.90 1.04	8.85 .888	9.58 .795	11.3 .640	v M
2.450 10.5	.415	2 7	7 11	13 14	1.45 1.19	1.86 1.31	2.07 1.38	2.40 1.50	2.70 1.60	3.07 1.72	3.55 1.83	4.60 1.84	5.86 1.55	6.62 1.34	7.33 1.17	8.02 1.03	8.92 .880	9.63 .791	11.4 .639	v M
2.397 11.0	.446	3 8	8 12	14 15	1.79 1.11	2.21 1.21	2.43 1.26	2.77 1.36	3.09 1.46	3.48 1.57	3.99 1.69	5.12 1.75	6.52 1.52	7.40 1.32	8.25 1.15	9.09 1.01	10.2 .858	11.1 .770	13.1 .626	v M
2.332 11.8	.442	3 8	9 13	15 16	1.89 1.13	2.37 1.23	2.61 1.29	2.98 1.39	3.32 1.48	3.73 1.59	4.27 1.70	5.45 1.75	6.86 1.51	7.74 1.31	8.56 1.14	9.36 .999	10.4 .854	11.2 .768	13.1 .627	v M
2.270 12.6	.428	3 8	9 14	16 17	1.94 1.14	2.46 1.25	2.72 1.32	3.13 1.44	3.48 1.54	3.92 1.66	4.47 1.77	5.66 1.81	7.06 1.55	7.91 1.34	8.69 1.16	9.46 1.02	10.5 .867	11.3 .778	13.2 .630	v M
2.195 13.7	.402	3 9	9 14	16 17	2.23 1.20	2.68 1.32	2.92 1.39	3.29 1.51	3.62 1.61	4.03 1.74	4.56 1.87	5.72 1.94	7.10 1.70	7.94 1.50	8.72 1.31	9.48 1.16	10.5 .993	11.3 .891	13.2 .719	v M

Table 6.

Three-stage sampling plans with $n_1 = n_2 = n_3$.

Poisson OC fractiles, $v = n_1 \lambda$, and $ASN = n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	Probability of Acceptance															
					.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
2.157 14.4	.396	3 9	9 14	17 18	2.29 1.21	2.79 1.35	3.04 1.43	3.43 1.55	3.78 1.67	4.20 1.80	4.75 1.92	5.93 1.95	7.31 1.68	8.14 1.47	8.90 1.29	9.63 1.14	10.6 .981	11.3 .884	13.2 .718	v M
2.120 15.0	.429	4 10	11 15	19 20	2.70 1.14	3.24 1.25	3.51 1.31	3.93 1.42	4.32 1.52	4.78 1.63	5.39 1.75	6.68 1.80	8.22 1.57	9.15 1.38	10.0 1.22	10.9 1.08	12.0 .935	12.8 .847	14.9 .697	v M
2.081 15.8	.428	4 10	12 16	20 21	2.83 1.16	3.42 1.27	3.72 1.34	4.17 1.45	4.58 1.54	5.06 1.65	5.69 1.76	7.03 1.80	8.59 1.56	9.52 1.36	10.4 1.20	11.2 1.07	12.3 .924	13.1 .839	15.0 .695	v M
2.044 16.6	.436	5 11	12 17	21 22	3.22 1.12	3.80 1.21	4.09 1.27	4.54 1.38	4.94 1.48	5.44 1.60	6.08 1.73	7.45 1.81	9.09 1.59	10.1 1.40	11.1 1.23	12.0 1.08	13.3 .928	14.2 .838	16.5 .690	v M
2.006 17.6	.429	5 11	13 18	22 23	3.35 1.13	3.98 1.23	4.29 1.30	4.77 1.40	5.19 1.50	5.71 1.62	6.37 1.75	7.77 1.82	9.41 1.59	10.4 1.40	11.3 1.22	12.2 1.08	13.4 .929	14.4 .840	16.5 .691	v M
1.968 18.6	.417	5 11	13 19	23 24	3.42 1.14	4.10 1.26	4.44 1.33	4.94 1.45	5.39 1.56	5.92 1.69	6.59 1.82	8.00 1.88	9.63 1.63	10.6 1.42	11.5 1.24	12.4 1.10	13.5 .941	14.4 .848	16.5 .695	v M
1.931 19.7	.431	6 12	14 20	25 26	3.92 1.11	4.62 1.22	4.97 1.28	5.50 1.40	5.96 1.51	6.52 1.63	7.22 1.76	8.71 1.83	10.5 1.59	11.5 1.39	12.5 1.22	13.5 1.07	14.8 .920	15.8 .830	18.1 .686	v M
1.911 20.4	.430	6 12	15 21	26 27	3.98 1.11	4.75 1.22	5.13 1.29	5.69 1.41	6.18 1.51	6.76 1.64	7.49 1.76	9.01 1.82	10.8 1.58	11.8 1.38	12.8 1.21	13.7 1.07	14.9 .919	15.9 .831	18.1 .688	v M
1.885 21.3	.421	6 13	16 21	26 27	4.34 1.15	5.03 1.26	5.38 1.33	5.90 1.43	6.36 1.53	6.93 1.65	7.65 1.78	9.17 1.86	10.9 1.65	12.0 1.46	13.0 1.29	13.9 1.15	15.1 .998	16.0 .906	18.2 .751	v M
1.853 22.5	.407	6 13	16 22	27 28	4.45 1.17	5.19 1.29	5.55 1.36	6.09 1.48	6.57 1.59	7.14 1.72	7.87 1.85	9.40 1.92	11.1 1.69	12.2 1.49	13.1 1.32	14.0 1.17	15.2 1.01	16.1 .916	18.2 .756	v M
1.825 23.7	.418	7 14	17 23	29 30	4.95 1.14	5.71 1.25	6.08 1.31	6.64 1.43	7.14 1.54	7.74 1.66	8.50 1.80	10.1 1.88	11.9 1.67	13.0 1.47	14.0 1.30	15.0 1.15	16.3 .995	17.3 .901	19.7 .745	v M
1.799 24.8	.407	7 14	17 24	30 31	5.06 1.15	5.87 1.27	6.26 1.35	6.84 1.48	7.36 1.59	7.97 1.73	8.74 1.86	10.3 1.94	12.2 1.70	13.2 1.50	14.2 1.32	15.2 1.17	16.4 1.01	17.4 .909	19.7 .749	v M
1.771 26.2	.426	8 15	20 26	33 34	5.68 1.13	6.58 1.23	7.01 1.30	7.65 1.41	8.21 1.52	8.87 1.64	9.70 1.76	11.4 1.84	13.4 1.62	14.5 1.43	15.6 1.26	16.6 1.12	17.9 .971	18.9 .884	21.3 .738	v M
1.747 27.6	.418	8 16	21 26	34 35	6.08 1.17	6.93 1.28	7.35 1.35	7.97 1.45	8.53 1.56	9.19 1.67	10.0 1.80	11.8 1.87	13.7 1.66	14.9 1.47	15.9 1.31	16.9 1.18	18.2 1.03	19.1 .944	21.4 .793	v M
1.731 28.5	.425	9 16	21 28	36 37	6.30 1.11	7.26 1.22	7.73 1.29	8.41 1.41	9.00 1.52	9.70 1.65	10.6 1.78	12.4 1.86	14.4 1.63	15.6 1.43	16.7 1.26	17.7 1.12	19.1 .968	20.2 .880	22.7 .735	v M
1.715 29.5	.426	9 17	23 28	37 38	6.73 1.15	7.65 1.26	8.10 1.32	8.77 1.43	9.36 1.53	10.1 1.64	10.9 1.76	12.8 1.84	14.8 1.63	16.1 1.45	17.2 1.29	18.2 1.16	19.5 1.02	20.5 .931	22.9 .785	v M

Table 7.

Seven-stage sampling plans with $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7$.

Poisson OC fractiles, $v = n_1 \lambda$, and $ASN = n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	a_4 r_4	a_5 r_5	a_6 r_6	a_7 r_7	Probability of Acceptance															
									.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
20.59 .391	.267	-1 1	-1 2	0 2	0 2	0 2	0 2	1 2	.001 3.01	.005 3.03	.009 3.05	.021 3.10	.038 3.16	.068 3.22	.121 3.23	.287 2.89	.573 2.07	.789 1.60	1.01 1.25	1.24 1.00	1.54 .777	1.77 .657	2.30 .477	v M
10.66 1.04	.316	-1 2	0 2	0 2	1 3	1 3	1 3	2 3	.016 2.06	.036 2.13	.051 2.18	.081 2.28	.117 2.37	.172 2.46	.260 2.54	.518 2.38	.937 1.83	1.25 1.49	1.56 1.23	1.89 1.04	2.32 .854	2.66 .748	3.46 .578	v M
8.057 1.50	.276	-1 2	0 2	0 3	1 4	1 4	2 4	3 4	.023 2.09	.050 2.20	.071 2.28	.112 2.42	.158 2.56	.224 2.72	.323 2.86	.586 2.75	.983 2.09	1.27 1.66	1.58 1.33	1.89 1.10	2.33 .876	2.66 .760	3.46 .580	v M
6.579 1.97	.243	-1 2	0 3	0 3	1 4	1 4	2 4	3 4	.039 2.16	.078 2.33	.103 2.44	.148 2.63	.195 2.81	.261 3.01	.358 3.20	.613 3.14	.999 2.45	1.28 1.96	1.58 1.57	1.89 1.28	2.33 .996	2.66 .845	3.46 .619	v M
5.478 2.54	.227	-1 2	0 3	0 4	1 5	2 5	3 5	4 5	.044 2.18	.095 2.38	.129 2.51	.190 2.74	.252 2.95	.332 3.17	.446 3.38	.724 3.34	1.11 2.63	1.38 2.12	1.65 1.70	1.94 1.37	2.35 1.05	2.67 .876	3.46 .626	v M
4.722 3.16	.233	-1 3	0 3	1 4	2 5	3 6	4 6	5 6	.092 2.20	.157 2.35	.198 2.45	.269 2.62	.341 2.78	.435 2.97	.568 3.13	.888 3.09	1.32 2.49	1.61 2.07	1.89 1.74	2.17 1.48	2.55 1.23	2.85 1.08	3.57 .849	v M
4.153 3.85	.212	-1 3	0 4	1 4	2 5	3 6	4 7	6 7	.129 2.29	.203 2.49	.248 2.61	.323 2.82	.398 3.01	.496 3.23	.633 3.43	.955 3.39	1.38 2.76	1.65 2.32	1.92 1.96	2.19 1.68	2.56 1.39	2.85 1.22	3.57 .928	v M
3.842 4.37	.231	-1 3	0 4	2 5	3 6	4 7	5 8	7 8	.164 2.29	.264 2.46	.322 2.56	.417 2.72	.509 2.88	.627 3.05	.787 3.20	1.16 3.14	1.64 2.52	1.96 2.09	2.26 1.75	2.55 1.49	2.94 1.23	3.23 1.08	3.91 .843	v M
3.572 4.95	.241	-1 3	1 5	2 6	3 7	5 8	6 9	8 9	.188 2.07	.318 2.20	.393 2.30	.512 2.49	.621 2.67	.755 2.89	.933 3.10	1.34 3.14	1.87 2.54	2.22 2.09	2.57 1.71	2.92 1.42	3.40 1.13	3.77 .968	4.64 .722	v M
3.444 5.27	.237	-1 3	1 5	2 6	3 8	5 9	7 10	9 10	.188 2.07	.323 2.21	.405 2.32	.539 2.53	.661 2.75	.809 2.99	1.00 3.22	1.42 3.24	1.94 2.60	2.28 2.13	2.61 1.74	2.95 1.44	3.41 1.14	3.78 .973	4.64 .723	v M
3.271 5.78	.261	0 4	1 5	2 7	4 8	5 10	7 11	10 11	.338 1.46	.488 1.75	.570 1.92	.700 2.20	.820 2.46	.968 2.75	1.16 3.04	1.61 3.12	2.22 2.47	2.68 1.96	3.20 1.54	3.80 1.22	4.65 .936	5.33 .795	6.91 .591	v M
3.071 6.50	.255	0 4	1 6	3 7	5 9	6 10	8 12	11 12	.394 1.50	.566 1.77	.657 1.93	.798 2.19	.927 2.42	1.09 2.69	1.30 2.95	1.77 3.05	2.40 2.48	2.85 2.02	3.33 1.62	3.87 1.30	4.68 .997	5.34 .835	6.92 .603	v M
2.922 7.17	.243	0 4	1 6	3 8	5 9	7 11	9 13	12 13	.407 1.52	.599 1.82	.701 2.00	.857 2.28	.998 2.53	1.17 2.81	1.39 3.08	1.87 3.17	2.49 2.57	2.92 2.09	3.37 1.68	3.89 1.34	4.68 1.01	5.34 .842	6.92 .604	v M
2.754 8.09	.247	0 5	2 7	4 8	6 10	8 12	10 14	13 14	.593 1.59	.780 1.82	.879 1.96	1.03 2.19	1.17 2.41	1.35 2.67	1.58 2.93	2.10 3.08	2.77 2.59	3.23 2.17	3.70 1.81	4.21 1.52	4.93 1.22	5.51 1.04	6.98 .767	v M
2.624 8.96	.246	0 5	2 7	5 9	7 11	9 13	12 15	15 16	.649 1.62	.872 1.86	.988 2.00	1.17 2.23	1.33 2.44	1.53 2.67	1.78 2.91	2.34 3.00	3.03 2.50	3.49 2.10	3.93 1.76	4.40 1.49	5.05 1.20	5.60 1.04	7.00 .766	v M
2.518 9.82	.234	0 5	2 7	5 10	7 12	10 14	12 16	16 17	.665 1.64	.911 1.91	1.04 2.07	1.24 2.33	1.41 2.56	1.62 2.83	1.88 3.10	2.44 3.20	3.12 2.65	3.55 2.21	3.98 1.84	4.43 1.54	5.07 1.23	5.60 1.05	7.00 .768	v M
2.415 10.8	.237	0 5	3 8	5 11	8 13	11 16	14 18	18 19	.723 1.59	1.02 1.85	1.17 2.01	1.40 2.29	1.60 2.54	1.83 2.82	2.12 3.10	2.71 3.22	3.42 2.66	3.87 2.21	4.31 1.83	4.77 1.52	5.42 1.21	5.94 1.03	7.24 .762	v M
2.319 11.9	.230	0 6	3 8	6 11	9 14	12 17	15 19	19 20	.920 1.73	1.20 1.98	1.34 2.12	1.56 2.36	1.75 2.59	1.98 2.84	2.27 3.11	2.89 3.22	3.61 2.71	4.07 2.29	4.50 1.93	4.94 1.65	5.54 1.36	6.03 1.19	7.27 .908	v M
2.234 13.1	.249	1 6	4 10	7 13	10 16	13 19	17 22	22 23	1.09 1.37	1.46 1.64	1.64 1.82	1.90 2.11	2.12 2.37	2.38 2.68	2.69 3.00	3.36 3.18	4.18 2.64	4.74 2.17	5.32 1.76	5.95 1.42	6.85 1.10	7.56 .936	9.27 .692	v M
2.163 14.2	.243	1 7	4 10	8 13	11 17	15 20	19 23	23 24	1.33 1.52	1.66 1.78	1.83 1.93	2.07 2.18	2.29 2.41	2.55 2.67	2.87 2.96	3.56 3.13	4.40 2.65	4.95 2.23	5.50 1.86	6.08 1.57	6.92 1.26	7.60 1.09	9.28 .816	v M
2.113 15.2	.248	1 7	5 11	9 14	12 18	16 22	21 25	25 26	1.44 1.50	1.82 1.75	2.01 1.89	2.28 2.13	2.52 2.36	2.79 2.62	3.14 2.90	3.87 3.07	4.75 2.60	5.32 2.19	5.88 1.83	6.45 1.54	7.24 1.25	7.86 1.09	9.41 .822	v M
2.057 16.3	.238	1 7	5 11	9 15	13 19	17 23	22 26	27 28	1.46 1.51	1.88 1.78	2.09 1.94	2.39 2.20	2.65 2.45	2.94 2.73	3.30 3.02	4.04 3.18	4.90 2.67	5.44 2.25	5.97 1.88	6.50 1.58	7.26 1.27	7.87 1.10	9.41 .824	v M

Table 7.

Seven-stage sampling plans with $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7$.

Poisson OC fractiles, $v = n_1\lambda$, and $ASN = n_1M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	a_4 r_4	a_5 r_5	a_6 r_6	a_7 r_7	Probability of Acceptance															
									.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
2.015 17.3	.237	1 7	5 12	10 16	14 20	18 24	23 27	29 30	1.51 1.53	1.97 1.81	2.20 1.98	2.54 2.25	2.82 2.50	3.13 2.79	3.51 3.07	4.27 3.22	5.15 2.72	5.68 2.29	6.19 1.93	6.70 1.63	7.41 1.32	7.98 1.14	9.45 .843	v M
1.999 17.8	.256	2 8	6 13	11 17	15 21	19 25	24 29	30 31	1.90 1.38	2.35 1.64	2.57 1.79	2.87 2.04	3.13 2.28	3.43 2.56	3.81 2.86	4.61 3.07	5.60 2.61	6.25 2.18	6.92 1.80	7.63 1.50	8.61 1.19	9.40 1.02	11.3 .769	v M
1.970 18.6	.248	2 8	6 13	11 17	15 22	19 26	25 30	31 32	1.91 1.39	2.38 1.66	2.60 1.82	2.93 2.09	3.20 2.35	3.51 2.64	3.89 2.95	4.69 3.16	5.66 2.67	6.29 2.22	6.94 1.83	7.63 1.51	8.62 1.19	9.40 1.02	11.3 .769	v M
1.932 19.7	.249	2 9	7 13	12 18	16 23	21 28	27 32	33 34	2.17 1.45	2.62 1.69	2.84 1.83	3.16 2.08	3.44 2.32	3.77 2.59	4.17 2.89	5.00 3.09	6.00 2.63	6.65 2.21	7.29 1.86	7.95 1.57	8.87 1.28	9.59 1.12	11.3 .868	v M
1.900 20.7	.241	2 9	7 14	12 18	17 23	22 28	27 32	34 35	2.27 1.50	2.73 1.75	2.94 1.90	3.26 2.14	3.54 2.38	3.86 2.65	4.26 2.94	5.10 3.14	6.09 2.69	6.72 2.29	7.34 1.94	7.99 1.65	8.88 1.34	9.60 1.17	11.3 .888	v M
1.874 21.7	.240	2 9	7 14	13 19	18 24	23 30	29 34	36 37	2.32 1.51	2.82 1.77	3.06 1.92	3.41 2.17	3.71 2.41	4.06 2.68	4.48 2.97	5.34 3.14	6.34 2.67	6.95 2.27	7.55 1.92	8.15 1.63	8.99 1.34	9.67 1.17	11.4 .888	v M
1.851 22.5	.241	2 9	8 15	13 20	19 25	24 31	30 35	37 38	2.40 1.49	2.95 1.75	3.21 1.91	3.58 2.16	3.89 2.40	4.24 2.67	4.67 2.96	5.54 3.16	6.56 2.71	7.20 2.29	7.82 1.93	8.43 1.64	9.28 1.33	9.94 1.16	11.6 .887	v M
1.835 23.2	.240	2 9	8 15	14 21	19 26	25 32	31 36	39 40	2.41 1.49	2.99 1.75	3.28 1.91	3.68 2.18	4.02 2.42	4.39 2.71	4.84 3.01	5.73 3.19	6.74 2.71	7.37 2.29	7.96 1.93	8.56 1.63	9.37 1.33	10.0 1.16	11.6 .887	v M
1.809 24.4	.241	3 10	8 16	14 21	20 27	26 33	32 37	40 41	2.81 1.43	3.35 1.70	3.60 1.87	3.96 2.13	4.27 2.39	4.62 2.68	5.06 2.99	5.96 3.21	7.02 2.74	7.72 2.31	8.42 1.93	9.16 1.61	10.2 1.29	11.1 1.11	13.1 .841	v M
1.790 25.3	.244	3 10	9 16	15 23	21 29	27 35	34 39	42 43	2.85 1.39	3.46 1.65	3.75 1.81	4.15 2.09	4.49 2.35	4.87 2.65	5.33 2.97	6.26 3.20	7.34 2.74	8.04 2.30	8.72 1.91	9.44 1.58	10.4 1.27	11.2 1.10	13.2 .837	v M
1.777 25.9	.243	3 10	9 17	16 23	22 29	28 36	35 41	43 44	2.91 1.40	3.55 1.67	3.86 1.84	4.28 2.10	4.62 2.35	5.01 2.64	5.48 2.95	6.43 3.18	7.52 2.73	8.21 2.30	8.88 1.92	9.57 1.61	10.5 1.30	11.3 1.12	13.2 .850	v M
1.758 26.9	.243	3 11	10 17	16 23	23 30	29 36	36 42	45 46	3.24 1.48	3.81 1.73	4.08 1.87	4.48 2.12	4.82 2.36	5.21 2.63	5.68 2.93	6.65 3.13	7.77 2.68	8.47 2.28	9.15 1.94	9.84 1.66	10.8 1.37	11.5 1.20	13.3 .936	v M
1.745 27.7	.241	3 11	10 17	17 24	23 31	30 37	37 43	46 47	3.27 1.49	3.87 1.74	4.16 1.89	4.58 2.14	4.93 2.38	5.33 2.66	5.82 2.96	6.80 3.16	7.92 2.71	8.61 2.30	9.27 1.95	9.94 1.66	10.9 1.37	11.6 1.20	13.3 .937	v M
1.734 28.3	.239	3 11	10 17	17 25	24 32	31 38	38 44	47 48	3.28 1.49	3.91 1.75	4.21 1.90	4.66 2.16	5.02 2.41	5.44 2.70	5.94 3.00	6.92 3.21	8.03 2.75	8.71 2.34	9.35 1.98	10.0 1.68	10.9 1.38	11.6 1.21	13.3 .938	v M
1.719 29.2	.238	3 11	10 18	18 25	25 32	32 39	39 45	48 49	3.39 1.52	4.05 1.79	4.37 1.95	4.81 2.20	5.18 2.44	5.60 2.71	6.10 2.99	7.10 3.18	8.23 2.73	8.91 2.33	9.55 1.98	10.2 1.70	11.0 1.41	11.7 1.24	13.4 .955	v M

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Appendix

Tables of two-, three- and seven-stage sampling plans for $\alpha = 0.005$
and $\beta = 0.010$.

Tables of multiple sampling plans from Military Standard 105D.

Table A 1.

Double sampling plans with $n_1 = n_2$.

Poisson OC fractiles, $v = n_1 \lambda$, and ASN = $n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.005$ and $\beta = 0.010$. $R = \lambda_{.010} / \lambda_{.995}$

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	Probability of Acceptance															
				.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
3.300 16.8	.575	7 13	19 20	4.31 1.07	5.06 1.14	5.44 1.18	6.04 1.25	6.58 1.31	7.24 1.39	8.10 1.46	9.93 1.47	12.0 1.29	13.3 1.15	14.4 1.03	15.4 .938	16.7 .836	17.7 .775	19.9 .669	v M
3.238 17.4	.572	7 13	20 21	4.46 1.08	5.26 1.16	5.67 1.21	6.30 1.28	6.87 1.35	7.56 1.42	8.44 1.49	10.3 1.47	12.4 1.27	13.7 1.13	14.8 1.01	15.8 .918	17.0 .821	18.0 .763	20.1 .663	v M
3.144 18.3	.583	8 14	21 22	4.92 1.06	5.74 1.12	6.16 1.16	6.80 1.23	7.38 1.30	8.09 1.37	9.00 1.44	10.9 1.45	13.2 1.28	14.4 1.14	15.6 1.03	16.7 .931	18.0 .832	19.1 .772	21.4 .669	v M
3.061 19.2	.546	7 14	21 22	4.91 1.12	5.71 1.21	6.13 1.27	6.76 1.35	7.34 1.43	8.03 1.50	8.93 1.56	10.8 1.54	13.0 1.34	14.2 1.19	15.3 1.07	16.3 .975	17.5 .874	18.4 .814	20.4 .709	v M
2.985 20.1	.555	8 15	22 23	5.34 1.09	6.17 1.17	6.59 1.21	7.24 1.29	7.83 1.36	8.54 1.44	9.46 1.51	11.4 1.52	13.6 1.35	14.9 1.21	16.0 1.09	17.1 .993	18.4 .889	19.4 .826	21.6 .717	v M
2.935 20.7	.553	8 15	23 24	5.53 1.11	6.40 1.19	6.85 1.25	7.53 1.33	8.14 1.40	8.88 1.48	9.83 1.54	11.8 1.53	14.1 1.33	15.3 1.18	16.5 1.07	17.5 .970	18.8 .871	19.7 .811	21.8 .708	v M
2.869 21.6	.561	9 16	24 25	5.99 1.08	6.87 1.15	7.33 1.20	8.02 1.27	8.64 1.34	9.40 1.42	10.4 1.49	12.4 1.51	14.7 1.34	16.1 1.20	17.3 1.08	18.4 .986	19.7 .884	20.7 .823	23.0 .715	v M
2.828 22.2	.560	9 16	25 26	6.16 1.09	7.10 1.18	7.58 1.23	8.30 1.31	8.95 1.38	9.73 1.45	10.7 1.52	12.8 1.51	15.2 1.32	16.5 1.18	17.7 1.06	18.7 .965	20.1 .868	21.1 .809	23.2 .708	v M
2.770 23.2	.568	10 17	26 27	6.64 1.07	7.59 1.14	8.07 1.19	8.81 1.26	9.47 1.33	10.3 1.40	11.3 1.48	13.4 1.49	15.8 1.33	17.2 1.19	18.5 1.08	19.6 .980	21.0 .880	22.1 .819	24.5 .714	v M
2.737 23.7	.566	10 17	27 28	6.80 1.08	7.81 1.16	8.31 1.21	9.08 1.29	9.77 1.36	10.6 1.44	11.6 1.50	13.8 1.50	16.2 1.31	17.6 1.17	18.9 1.05	20.0 .961	21.4 .865	22.4 .807	24.7 .708	v M
2.685 24.6	.574	11 18	28 29	7.30 1.07	8.31 1.13	8.82 1.17	9.60 1.25	10.3 1.31	11.1 1.39	12.2 1.46	14.4 1.48	16.9 1.32	18.4 1.18	19.7 1.07	20.8 .975	22.3 .876	23.4 .816	25.9 .713	v M
2.634 25.6	.544	10 18	28 29	7.29 1.12	8.29 1.21	8.79 1.26	9.56 1.35	10.2 1.42	11.1 1.50	12.1 1.57	14.3 1.56	16.8 1.37	18.1 1.23	19.4 1.11	20.5 1.01	21.8 .910	22.8 .850	25.0 .746	v M
2.590 26.6	.551	11 19	29 30	7.75 1.09	8.76 1.17	9.27 1.22	10.0 1.30	10.7 1.37	11.6 1.45	12.7 1.52	14.9 1.54	17.4 1.38	18.8 1.24	20.1 1.13	21.3 1.03	22.7 .925	23.7 .863	26.1 .754	v M
2.561 27.2	.549	11 19	30 31	7.96 1.11	9.01 1.20	9.55 1.25	10.4 1.33	11.1 1.41	11.9 1.48	13.0 1.55	15.3 1.55	17.8 1.36	19.3 1.22	20.5 1.10	21.7 1.01	23.1 .907	24.1 .848	26.4 .746	v M
2.522 28.1	.556	12 20	31 32	8.44 1.09	9.50 1.16	10.0 1.21	10.9 1.28	11.6 1.36	12.5 1.43	13.6 1.51	15.9 1.53	18.5 1.37	20.0 1.23	21.3 1.12	22.5 1.02	24.0 .920	25.0 .859	27.5 .752	v M
2.497 28.8	.555	12 20	32 33	8.63 1.10	9.75 1.18	10.3 1.23	11.2 1.32	11.9 1.39	12.8 1.47	14.0 1.54	16.3 1.54	18.9 1.35	20.4 1.21	21.7 1.10	22.9 1.00	24.3 .904	25.4 .846	27.7 .745	v M
2.461 29.7	.561	13 21	33 34	9.13 1.08	10.3 1.15	10.8 1.20	11.7 1.27	12.4 1.34	13.3 1.42	14.5 1.50	16.9 1.52	19.6 1.36	21.1 1.23	22.5 1.11	23.7 1.02	25.2 .916	26.3 .856	28.9 .751	v M

Table A.2.

Three-stage sampling plans with $n_1 = n_2 = n_3$.

Poisson OC fractiles, $v = n_1 \lambda$, and ASN = $n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.005$ and $\beta = 0.010$. $R = \lambda_{.010} / \lambda_{.995}$

R a_0	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	Probability of Acceptance															
					.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
67.64 .962	.377	-1 2	0 2	1 2	.016 2.03	.037 2.07	.053 2.09	.086 2.13	.126 2.17	.189 2.20	.293 2.20	.599 2.00	1.08 1.56	1.41 1.30	1.74 1.11	2.06 .953	2.48 .801	2.80 .712	3.55 .563	v M
47.23 1.25	.657	0 2	1 3	3 4	.044 1.04	.099 1.10	.139 1.13	.219 1.20	.310 1.26	.442 1.33	.647 1.38	1.19 1.29	2.01 .978	2.59 .788	3.18 .645	3.80 .537	4.66 .435	5.33 .378	6.91 .290	v M
27.19 1.87	.570	0 3	1 3	3 4	.098 1.10	.172 1.19	.220 1.24	.309 1.33	.404 1.41	.539 1.51	.741 1.59	1.28 1.55	2.07 1.26	2.63 1.06	3.21 .900	3.82 .770	4.66 .638	5.33 .560	6.92 .433	v M
19.90 2.38	.540	0 3	2 4	3 4	.149 1.14	.245 1.22	.305 1.27	.412 1.35	.525 1.42	.680 1.50	.911 1.57	1.51 1.54	2.36 1.26	2.95 1.06	3.52 .896	4.10 .766	4.88 .635	5.49 .559	6.98 .434	v M
17.47 2.64	.518	0 3	2 4	4 5	.169 1.16	.280 1.25	.350 1.31	.472 1.40	.597 1.48	.767 1.56	1.01 1.62	1.62 1.56	2.47 1.25	3.03 1.05	3.58 .890	4.14 .762	4.90 .634	5.50 .559	6.98 .433	v M
15.28 2.95	.501	0 3	2 5	5 6	.188 1.18	.327 1.29	.413 1.36	.564 1.48	.715 1.57	.913 1.67	1.19 1.75	1.84 1.67	2.69 1.32	3.24 1.09	3.76 .918	4.28 .783	4.99 .647	5.56 .568	6.99 .437	v M
10.67 4.03	.430	0 4	2 5	5 6	.323 1.30	.470 1.43	.555 1.50	.699 1.62	.842 1.73	1.03 1.84	1.30 1.93	1.93 1.89	2.76 1.57	3.29 1.34	3.80 1.15	4.31 1.00	5.01 .847	5.57 .751	7.00 .583	v M
8.963 4.76	.410	0 4	2 6	6 7	.397 1.37	.583 1.53	.687 1.63	.859 1.77	1.02 1.88	1.24 2.00	1.53 2.08	2.21 2.00	3.06 1.61	3.60 1.37	4.09 1.17	4.58 1.02	5.22 .855	5.73 .759	7.05 .589	v M
8.062 5.29	.496	1 5	3 7	7 8	.617 1.15	.834 1.26	.954 1.33	1.15 1.44	1.34 1.55	1.58 1.67	1.92 1.79	2.70 1.81	3.72 1.53	4.40 1.31	5.07 1.12	5.76 .961	6.72 .797	7.47 .702	9.24 .551	v M
7.194 5.96	.475	1 5	4 8	8 9	.697 1.16	.958 1.28	1.10 1.35	1.33 1.46	1.54 1.56	1.81 1.68	2.17 1.79	3.00 1.81	4.05 1.52	4.72 1.30	5.37 1.11	6.01 .959	6.89 .800	7.59 .706	9.28 .553	v M
6.914 6.22	.469	1 5	4 9	9 10	.720 1.17	1.01 1.30	1.17 1.38	1.42 1.51	1.66 1.62	1.95 1.74	2.34 1.84	3.20 1.81	4.26 1.48	4.92 1.26	5.54 1.08	6.15 .937	6.99 .788	7.65 .700	9.29 .553	v M
6.141 7.11	.446	1 6	5 9	9 10	.901 1.24	1.17 1.35	1.32 1.42	1.56 1.52	1.79 1.62	2.07 1.72	2.46 1.82	3.34 1.84	4.43 1.57	5.12 1.37	5.75 1.20	6.38 1.06	7.20 .907	7.85 .816	9.41 .657	v M
5.531 8.04	.420	1 6	5 10	10 11	1.02 1.28	1.33 1.42	1.49 1.50	1.75 1.62	1.99 1.73	2.30 1.85	2.70 1.95	3.60 1.94	4.68 1.64	5.35 1.42	5.96 1.23	6.55 1.08	7.33 .926	7.94 .831	9.44 .665	v M
5.255 8.57	.414	1 6	5 11	11 12	1.08 1.31	1.43 1.48	1.62 1.57	1.91 1.71	2.18 1.83	2.50 1.95	2.93 2.05	3.86 2.01	4.96 1.66	5.63 1.43	6.23 1.24	6.80 1.09	7.54 .928	8.10 .834	9.51 .668	v M
4.897 9.37	.450	2 7	6 11	12 13	1.40 1.19	1.77 1.31	1.96 1.39	2.26 1.51	2.54 1.63	2.88 1.75	3.33 1.88	4.33 1.92	5.55 1.65	6.30 1.43	7.01 1.24	7.72 1.08	8.68 .914	9.43 .813	11.3 .645	v M
4.794 9.63	.454	2 7	7 11	13 14	1.45 1.19	1.86 1.31	2.07 1.38	2.40 1.50	2.70 1.60	3.07 1.72	3.55 1.83	4.60 1.84	5.86 1.55	6.62 1.34	7.33 1.17	8.02 1.03	8.92 .880	9.63 .791	11.4 .639	v M
4.480 10.6	.444	2 8	8 11	14 15	1.67 1.24	2.07 1.36	2.28 1.43	2.62 1.54	2.93 1.63	3.31 1.73	3.82 1.83	4.91 1.84	6.21 1.58	7.00 1.39	7.72 1.23	8.40 1.10	9.28 .956	9.95 .871	11.6 .721	v M
4.243 11.4	.423	2 8	8 12	14 15	1.78 1.27	2.19 1.40	2.41 1.47	2.74 1.59	3.05 1.69	3.43 1.80	3.92 1.91	5.00 1.93	6.29 1.67	7.06 1.46	7.77 1.29	8.44 1.14	9.31 .986	9.97 .893	11.6 .730	v M
4.044 12.2	.410	2 8	8 13	15 16	1.88 1.30	2.34 1.45	2.57 1.53	2.93 1.66	3.25 1.77	3.65 1.89	4.16 2.00	5.26 2.01	6.54 1.71	7.30 1.49	7.98 1.31	8.63 1.16	9.46 .998	10.1 .902	11.6 .736	v M
3.855 13.1	.445	3 9	10 14	17 18	2.30 1.21	2.80 1.33	3.06 1.40	3.45 1.51	3.81 1.61	4.24 1.72	4.81 1.83	6.02 1.86	7.44 1.61	8.30 1.41	9.08 1.24	9.83 1.10	10.8 .951	11.5 .862	13.3 .709	v M
3.690 14.0	.429	3 9	10 16	18 19	2.40 1.23	2.96 1.37	3.24 1.45	3.66 1.58	4.04 1.69	4.49 1.82	5.07 1.94	6.29 1.97	7.70 1.70	8.53 1.48	9.29 1.30	10.0 1.15	10.9 .984	11.6 .887	13.3 .720	v M
3.584 14.7	.451	4 10	10 16	19 20	2.77 1.17	3.31 1.29	3.58 1.37	4.00 1.49	4.37 1.61	4.83 1.74	5.42 1.88	6.68 1.94	8.17 1.69	9.07 1.48	9.92 1.30	10.7 1.14	11.9 .972	12.7 .871	14.8 .705	v M

Table A 2.

Three-stage sampling plans with $n_1 = n_2 = n_3$.

Poisson OC fractiles, $v = n_1 \lambda$, and ASN = $n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.005$ and $\beta = 0.010$. $R = \lambda_{.010} / \lambda_{.995}$

R	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	Probability of Acceptance															
					.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
3.537	.452	4	11	20	2.83	3.41	3.70	4.15	4.54	5.02	5.64	6.94	8.45	9.36	10.2	11.0	12.1	12.9	14.9	v
15.0		10	16	21	1.17	1.29	1.36	1.48	1.60	1.72	1.84	1.88	1.62	1.42	1.24	1.10	.949	.857	.701	M
3.423	.440	4	11	21	2.92	3.57	3.89	4.37	4.79	5.28	5.92	7.24	8.74	9.63	10.4	11.2	12.2	13.0	14.9	v
15.8		10	18	22	1.18	1.33	1.42	1.56	1.68	1.82	1.95	1.98	1.70	1.48	1.29	1.14	.974	.876	.710	M
3.278	.422	4	12	21	3.20	3.77	4.06	4.50	4.90	5.38	6.01	7.32	8.85	9.75	10.6	11.4	12.4	13.1	15.0	v
17.0		11	17	22	1.23	1.36	1.44	1.56	1.67	1.79	1.91	1.97	1.72	1.52	1.35	1.20	1.05	.949	.780	M
3.173	.410	4	12	22	3.33	3.95	4.25	4.72	5.13	5.62	6.26	7.59	9.11	10.0	10.8	11.6	12.5	13.3	15.1	v
18.0		11	18	23	1.26	1.41	1.49	1.62	1.74	1.87	2.00	2.03	1.76	1.55	1.36	1.21	1.05	.957	.785	M
3.060	.430	5	14	24	3.85	4.51	4.83	5.32	5.76	6.29	6.96	8.38	10.0	11.0	11.9	12.7	13.8	14.6	16.6	v
19.2		12	20	25	1.20	1.33	1.40	1.52	1.64	1.76	1.90	1.97	1.74	1.53	1.35	1.20	1.04	.939	.769	M
2.996	.422	5	14	25	3.94	4.65	4.99	5.51	5.96	6.51	7.19	8.63	10.3	11.2	12.1	12.9	13.9	14.7	16.7	v
19.9		12	21	26	1.22	1.36	1.44	1.58	1.70	1.83	1.96	2.02	1.77	1.55	1.37	1.21	1.05	.947	.774	M
2.915	.406	5	14	25	4.15	4.78	5.09	5.58	6.02	6.54	7.22	8.64	10.3	11.2	12.1	12.9	13.9	14.7	16.7	v
21.0		13	20	26	1.26	1.39	1.47	1.60	1.72	1.86	1.99	2.06	1.82	1.62	1.44	1.29	1.13	1.03	.845	M
2.858	.404	5	15	26	4.32	4.99	5.33	5.84	6.29	6.83	7.53	8.98	10.6	11.6	12.4	13.2	14.3	15.0	16.9	v
21.8		13	21	27	1.28	1.42	1.50	1.63	1.74	1.87	1.99	2.05	1.80	1.60	1.42	1.27	1.11	1.01	.839	M
2.792	.426	6	17	29	4.85	5.59	5.95	6.51	7.01	7.60	8.35	9.92	11.7	12.7	13.6	14.5	15.6	16.4	18.4	v
22.8		14	22	30	1.22	1.35	1.43	1.55	1.66	1.78	1.90	1.94	1.70	1.51	1.35	1.21	1.06	.974	.815	M
2.726	.415	6	17	30	4.99	5.78	6.17	6.75	7.26	7.86	8.62	10.2	12.0	13.0	13.9	14.7	15.8	16.6	18.5	v
23.9		14	23	31	1.25	1.40	1.48	1.61	1.73	1.85	1.98	2.00	1.74	1.53	1.36	1.22	1.07	.979	.819	M
2.690	.430	7	18	31	5.42	6.19	6.57	7.15	7.66	8.27	9.05	10.7	12.5	13.6	14.5	15.5	16.7	17.6	19.8	v
24.6		15	24	32	1.19	1.32	1.39	1.52	1.63	1.76	1.89	1.97	1.74	1.55	1.38	1.23	1.07	.978	.811	M
2.632	.418	7	18	32	5.57	6.38	6.78	7.38	7.90	8.52	9.31	10.9	12.8	13.8	14.8	15.6	16.8	17.7	19.8	v
25.7		15	25	33	1.21	1.36	1.44	1.57	1.70	1.83	1.97	2.02	1.77	1.57	1.39	1.24	1.08	.985	.815	M
2.577	.433	8	20	34	6.14	6.99	7.40	8.02	8.56	9.21	10.0	11.7	13.6	14.8	15.8	16.7	18.0	19.0	21.3	v
26.8		16	27	35	1.18	1.30	1.38	1.50	1.62	1.75	1.89	1.98	1.76	1.56	1.39	1.23	1.07	.973	.804	M
2.538	.425	8	20	35	6.25	7.15	7.58	8.22	8.78	9.44	10.3	12.0	13.9	15.0	16.0	16.9	18.1	19.1	21.3	v
27.8		16	28	36	1.19	1.33	1.41	1.55	1.67	1.81	1.95	2.03	1.79	1.58	1.40	1.24	1.08	.979	.808	M
2.498	.439	9	21	37	6.79	7.70	8.14	8.80	9.37	10.1	10.9	12.7	14.7	15.8	16.9	17.9	19.2	20.3	22.7	v
28.7		17	29	38	1.16	1.29	1.37	1.50	1.63	1.77	1.91	1.99	1.76	1.56	1.38	1.23	1.06	.964	.796	M
2.473	.436	9	22	38	6.89	7.87	8.33	9.02	9.61	10.3	11.2	13.0	15.0	16.1	17.2	18.2	19.5	20.4	22.8	v
29.4		17	30	39	1.17	1.30	1.38	1.51	1.63	1.77	1.91	1.99	1.75	1.55	1.37	1.22	1.06	.962	.797	M

Table A 3.

Seven-stage sampling plans with $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7$.

Poisson OC fractiles, $v = n_1 \lambda$, and $ASN = n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.005$ and $\beta = 0.010$. $R = \lambda_{.010} / \lambda_{.995}$

R	n_1/n_0	a _i							Probability of Acceptance															
		r ₁	r ₂	r ₃	r ₄	r ₅	r ₆	r ₇	.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
3.562 14.8	.219	0	3	7	10	13	16	22	1.32	1.63	1.78	1.99	2.18	2.41	2.70	3.30	4.01	4.44	4.85	5.25	5.80	6.23	7.37	1.15
3.465 15.5	.213	0	3	7	10	13	17	23	1.35	1.68	1.84	2.07	2.27	2.51	2.80	3.41	4.11	4.53	4.92	5.31	5.83	6.25	7.37	1.16
3.397 16.0	.220	0	4	7	11	14	18	24	1.45	1.82	1.98	2.22	2.43	2.67	2.97	3.59	4.32	4.77	5.19	5.60	6.17	6.63	7.77	1.13
3.262 17.1	.216	0	4	8	12	15	19	25	1.62	1.95	2.11	2.34	2.55	2.79	3.10	3.75	4.50	4.96	5.39	5.80	6.36	6.79	7.87	1.25
3.210 17.6	.213	0	4	8	12	16	20	26	1.64	1.99	2.16	2.41	2.63	2.88	3.20	3.86	4.61	5.06	5.47	5.87	6.40	6.82	7.87	1.25
3.170 18.0	.211	0	4	8	12	16	21	27	1.66	2.03	2.21	2.48	2.70	2.96	3.29	3.96	4.71	5.15	5.55	5.93	6.45	6.85	7.88	1.26
3.060 19.2	.209	0	4	9	13	17	22	28	1.80	2.19	2.37	2.63	2.86	3.13	3.46	4.14	4.91	5.37	5.78	6.17	6.69	7.09	8.07	1.28
2.971 20.2	.220	1	5	9	14	19	24	30	2.10	2.48	2.66	2.92	3.15	3.42	3.77	4.47	5.29	5.78	6.24	6.71	7.37	7.93	9.42	1.16
2.913 21.0	.219	1	5	10	15	19	24	32	2.16	2.58	2.79	3.08	3.32	3.61	3.97	4.69	5.51	6.00	6.46	6.90	7.53	8.05	9.46	1.16
2.851 21.9	.220	1	6	10	15	20	25	33	2.29	2.73	2.94	3.23	3.47	3.76	4.12	4.85	5.69	6.19	6.66	7.14	7.79	8.33	9.72	1.16
2.815 22.4	.218	1	6	10	16	21	26	34	2.32	2.80	3.02	3.32	3.58	3.88	4.25	5.00	5.84	6.34	6.80	7.25	7.88	8.39	9.73	1.17
2.766 23.2	.217	1	6	11	17	22	27	35	2.39	2.91	3.14	3.46	3.72	4.02	4.40	5.16	6.02	6.52	6.99	7.44	8.04	8.53	9.79	1.21
2.726 23.9	.222	1	7	12	17	23	28	36	2.65	3.09	3.30	3.60	3.86	4.17	4.55	5.33	6.23	6.77	7.27	7.76	8.42	8.95	10.2	1.24
2.700 24.4	.219	1	7	12	17	23	29	37	2.66	3.13	3.35	3.67	3.94	4.25	4.64	5.43	6.32	6.84	7.33	7.80	8.45	8.96	10.2	1.24
2.668 25.0	.217	1	7	12	18	23	29	38	2.70	3.19	3.42	3.75	4.03	4.35	4.74	5.54	6.43	6.95	7.43	7.89	8.51	9.00	10.2	1.24
2.612 26.1	.213	1	7	13	18	24	30	39	2.80	3.32	3.55	3.88	4.16	4.49	4.89	5.69	6.59	7.11	7.59	8.05	8.66	9.14	10.3	1.28
2.584 26.7	.212	1	7	13	19	25	31	40	2.84	3.39	3.64	3.99	4.28	4.62	5.02	5.83	6.73	7.25	7.72	8.17	8.76	9.22	10.4	1.29
2.559 27.3	.224	2	8	14	20	26	32	41	3.20	3.68	3.91	4.23	4.51	4.83	5.24	6.07	7.02	7.60	8.14	8.68	9.43	10.0	11.6	1.20
2.537 27.8	.221	2	8	14	20	26	32	42	3.21	3.72	3.96	4.29	4.58	4.91	5.32	6.16	7.10	7.67	8.19	8.72	9.45	10.0	11.6	1.20
2.506 28.5	.218	2	8	14	21	27	33	43	3.25	3.80	4.05	4.40	4.70	5.04	5.45	6.30	7.25	7.81	8.32	8.83	9.53	10.1	11.6	1.20
2.470 29.4	.216	2	8	15	21	28	34	44	3.35	3.91	4.16	4.52	4.82	5.16	5.59	6.44	7.40	7.96	8.47	8.98	9.66	10.2	11.6	1.24
2.456 29.8	.215	2	8	15	21	28	35	45	3.36	3.95	4.21	4.59	4.90	5.25	5.68	6.54	7.49	8.05	8.55	9.04	9.70	10.2	11.6	1.24

Table A 4.

Seven-stage sampling plans with $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7$ from Military Standard 105D.

Poisson OC fractiles, $v = n_1 \lambda$, and ASN = $n_1 M$.

Equivalent single sampling plan (a_0, n_0) for $\alpha = 0.05$ and $\beta = 0.10$. $R = \lambda_{.10} / \lambda_{.95}$.

R	n_1/n_0	a_1 r_1	a_2 r_2	a_3 r_3	a_4 r_4	a_5 r_5	a_6 r_6	a_7 r_7	Probability of Acceptance															
									.999	.995	.990	.975	.950	.900	.800	.500	.200	.100	.050	.025	.010	.005	.001	
8.929 1.31	.211	-1 2	-1 2	0 2	0 3	1 3	1 3	2 3	.015 3.09	.033 3.18	.046 3.25	.072 3.37	.103 3.48	.148 3.59	.219 3.66	.416 3.40	.712 2.63	.917 2.16	1.12 1.80	1.33 1.53	1.60 1.26	1.82 1.11	2.33 .862	
6.230 2.12	.247	-1 2	0 3	0 3	1 4	2 4	3 5	4 5	.040 2.16	.082 2.32	.111 2.43	.163 2.61	.217 2.77	.293 2.94	.404 3.08	.683 2.95	1.08 2.29	1.35 1.86	1.64 1.52	1.93 1.25	2.34 .988	2.67 .842	3.46 .618	
4.674 3.21	.234	-1 3	0 3	1 4	2 5	3 6	4 6	6 7	.092 2.20	.159 2.36	.200 2.46	.274 2.63	.348 2.80	.446 2.99	.584 3.15	.910 3.08	1.34 2.47	1.63 2.05	1.90 1.73	2.18 1.47	2.55 1.23	2.85 1.08	3.57 .849	
3.244 5.87	.219	-1 4	1 5	2 6	3 7	5 8	7 9	9 10	.302 2.20	.423 2.38	.490 2.49	.597 2.70	.700 2.89	.830 3.11	1.01 3.32	1.41 3.34	1.93 2.76	2.27 2.34	2.60 1.98	2.94 1.69	3.41 1.39	3.78 1.22	4.64 .940	
2.891 7.32	.243	0 4	1 6	3 8	5 10	7 11	10 12	13 14	.409 1.52	.607 1.84	.713 2.02	.877 2.31	1.02 2.57	1.20 2.85	1.43 3.12	1.92 3.17	2.54 2.56	2.96 2.09	3.40 1.69	3.90 1.35	4.69 1.02	5.34 .843	6.92 .604	
2.516 9.84	.258	0 5	3 8	6 10	8 13	11 15	14 17	18 19	.717 1.57	.997 1.79	1.14 1.92	1.37 2.14	1.56 2.35	1.79 2.58	2.08 2.81	2.70 2.90	3.45 2.40	3.93 2.00	4.39 1.68	4.86 1.41	5.50 1.15	6.00 1.00	7.27 .755	
2.184 13.9	.256	1 7	4 10	8 13	12 17	17 20	21 23	25 26	1.33 1.51	1.67 1.77	1.84 1.91	2.11 2.15	2.34 2.36	2.62 2.60	2.97 2.83	3.71 2.91	4.57 2.45	5.11 2.09	5.63 1.78	6.17 1.52	6.96 1.25	7.61 1.08	9.28 .816	
1.893 21.0	.252	2 9	7 14	13 19	19 25	25 29	31 33	37 38	2.32 1.50	2.82 1.76	3.07 1.91	3.43 2.15	3.74 2.36	4.10 2.60	4.54 2.85	5.44 2.97	6.46 2.53	7.08 2.17	7.66 1.86	8.24 1.60	9.04 1.32	9.70 1.16	11.4 .888	
1.708 29.9	.252	4 12	11 19	19 27	27 34	36 40	45 47	53 54	3.84 1.43	4.50 1.69	4.82 1.84	5.27 2.09	5.66 2.31	6.10 2.57	6.63 2.83	7.71 2.99	8.93 2.57	9.66 2.21	10.4 1.89	11.1 1.63	12.1 1.34	12.9 1.17	14.9 .904	
1.538 46.4	.388	6 16	17 27	29 39	40 49	53 58	65 68	77 78	6.26 1.52	7.19 1.81	7.61 1.97	8.21 2.23	8.70 2.47	9.24 2.73	9.89 3.01	11.2 3.19	12.6 2.78	13.4 2.41	14.1 2.09	14.8 1.81	15.8 1.52	16.5 1.34	18.4 1.04	
Tightened inspection.																								
2.839 7.59	.265	0 4	2 7	4 9	6 11	9 12	12 14	14 15	.426 1.40	.657 1.65	.787 1.80	.990 2.07	1.17 2.31	1.37 2.57	1.64 2.83	2.19 2.91	2.87 2.39	3.31 1.97	3.76 1.61	4.24 1.31	4.94 1.02	5.52 .852	6.98 .610	
2.249 12.9	.234	0 6	3 9	7 12	10 15	14 17	18 20	21 22	1.04 1.81	1.35 2.07	1.51 2.21	1.75 2.44	1.96 2.64	2.20 2.87	2.52 3.09	3.17 3.14	3.94 2.64	4.40 2.25	4.83 1.93	5.24 1.66	5.80 1.39	6.24 1.22	7.37 .935	
1.990 18.0	.252	1 8	6 12	11 17	16 22	22 25	27 29	32 33	1.83 1.63	2.28 1.87	2.51 2.01	2.84 2.23	3.13 2.43	3.46 2.66	3.87 2.89	4.71 2.97	5.66 2.51	6.22 2.15	6.74 1.85	7.25 1.60	7.92 1.34	8.44 1.19	9.76 .932	
1.805 24.6	.271	3 10	10 17	17 24	24 31	32 37	40 43	48 49	2.92 1.37	3.60 1.60	3.94 1.74	4.43 1.99	4.84 2.21	5.30 2.47	5.84 2.73	6.90 2.86	8.05 2.41	8.74 2.04	9.38 1.73	10.0 1.48	10.9 1.23	11.6 1.08	13.3 .835	
1.574 41.8	.449	6 15	16 25	26 36	37 46	49 55	61 64	72 73	5.64 1.40	6.52 1.68	6.94 1.84	7.53 2.11	8.01 2.36	8.55 2.65	9.19 2.94	10.4 3.11	11.8 2.68	12.6 2.30	13.4 1.96	14.1 1.68	15.2 1.39	16.1 1.22	18.2 .941	