Holomorphic Day, November 16, 2012
Department of Mathematical Sciences
University of Copenhagen

Abstract

The purpose of the holomorphic day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant 10-083122 from The Danish Council for Independent Research | Natural Sciences

1 Schedule

The lectures take place at the premises of the Department of Mathematical Sciences, Universitetsparken 5, Copenhagen.

Arrival, coffee, tea: 9.45-10.15 in E 419 (Fourth floor)

Núria Fagella: 10.15-11.05 in Aud. 10
Newton's method and absorbing regions

Morten Risager: 11.15-12.05 in Aud. 10
A double Dirichlet series and quantum unique ergodicity of weight $1/2$ *Eisenstein series*

Lunch: 12.15-13.15

Alexei Venkov: 13.15-14.05 in Aud. 9
Congruence properties of induced representations for $\mathrm{PSL}(2, \mathbb{Z})$

Henrik Laurberg Pedersen: 14.15-15.05 in Aud. 9
Extending inverses of entire functions of genus 1 and 2 to Pick functions in the upper half plane

Coffee break: 15.05-15.30

David Sauzin: 15.30-16.20 in Aud. 9
On the obtention of Ecalle-Voronin invariants via Resurgence theory

Luna Lomonaco: 16.30-17.20 in Aud. 9
Parabolic-like mappings

Dinner: 18.30-

2 Abstracts

Núria Fagella, Associate Professor, Universitat de Barcelona: *Newton's method and absorbing regions*

Newton's method for polynomials or entire maps can be regarded as a dynamical system on the Riemann sphere or, respectively, on the complex plane. Understanding the topology of its Julia set gives results which are interesting both dynamically and numerically. We present here a recent result which states that the Julia set of Newton's methods is always connected or, equivalently, that its stable regions are simply connected. In the talk however, we shall concentrate mostly on the main tool used to prove this theorem, namely the existence of absorbing regions in Baker domains (components on which all iterates tend to infinity). Absorbing regions (domains which eventually attract all orbits) are known to exist for each type of Fatou component except, until now, for Baker domains. This result takes a much more general form and it is based on work of Cowen on holomorphic maps from the right half plane to itself with no fixed points.

Morten Risager, Associate Professor, University of Copenhagen: *A double Dirichlet series and quantum unique ergodicity of weight* $1/2$ *Eisenstein series*

An important problem of quantum chaos is to describe the behavior of eigenfunctions of Laplacians when the eigenvalue goes to infinity. In this talk we explain how the problem of quantum unique ergodicity for weight $1/2$ Eisenstein series naturally leads to the study of a double Dirichlet series defined with classical L-series, and Fourier coefficients of GL_2 automorphic forms. This series is a function in 2 complex variables. We discuss its properties – meromorphic continuation, functional equations and natural bounds – and explain what properties of this series would allow us to conclude quantum unique ergodicity for weight $1/2$ Eisenstein series.

This is a joint work with Nicole Raulf (Lille) and Yiannis Petridis (UCL, London)

Alexei Venkov, Professor, University of Aarhus: *Congruence properties of induced representations for* $\mathrm{PSL}(2,\mathbb{Z})$

We start with recalling some important analytic properties of the Selberg zeta function $Z(s)$. This function is defined for a discrete co-finite subgroup G of $\mathrm{PSL}(2,\mathbb{R})$ and for an unitary representation U of G. We take $G = \mathrm{PSL}(2,\mathbb{Z})$ and $U = U(t)$ be a set of representations induced from one parameter family of Selberg's representations $u(t)$ for some Hecke congruence subgroup H of $\mathrm{PSL}(2,\mathbb{Z})$ (see [1]), t belongs to the interval $[0,1]$.

To understand deep analytic properties of $Z(s) = Z(s;G;U)$ it is important to study arithmetic congruence properties of algebraic kernels $k(t) = \ker u(t)$ in

H and $K(t) = \ker U(t)$ in $\mathrm{PSL}(2,\mathbb{Z})$. In 1964 Morris Newman found all values of t for which $k(t)$ are congruence subgroups (see [N]).The problem of existence of non-congruence subgroups goes back to Felix Klein, Robert Fricke and George Pick from 19'th century (see [3] and [4]), but to find interesting explicit examples is not easy even up to now. To see some infinite sequenses of non-congruence arithmetic groups there is some remarkable geometric method of Zograf who applied this together with Selberg's congruence theorem to prove Rademacher's congruence conjecture (see [5]).

In the talk we recall the results mentioned above, and we reprove the Newman theorem for $K(t)$ and compare the method of the proof with Zograf's approach.

References

[1] A. Selberg, Remarks on the distribution of poles of Eisenstein series, Collected Papers, **2** 2,15–46, Springer 1988.

[2] M. Newman, On a problem of G. Sansone, Ann.Mat.Pura Appl.(4) **65**, 27–34

[3] R. Fricke, Ueber die Substitutionsgruppen, welche zu den aus dem Legendre'schen integral modul k(w) gezogenen Wurzeln gehören, Math.Annalen **28** (1887), 99–118.

[4] G. Pick, Ueber gewisse ganzzahlige lineare Substitutionen, welche sich nicht durch algebraische Congruenzen erkl´aren lassen, Math.Annalen **28** (1887), 119–124.

[5] P. Zograf, A spectral proof of Rademacher's conjecture for congruence subgroups of the modular group, J. Reine Angew. Math. **414**(1991), 113–116.

Henrik Laurberg Pedersen, Professor, University of Copenhagen: *Extending inverses of entire functions of genus 1 and 2 to Pick functions in the upper half plane*

Euler's gamma function increases on an interval of the form (α, ∞), where α is the unique point x in $(0, \infty)$ where $\Gamma'(x) = 0$. Recently, Uchiyama proved that the inverse of Γ defined on $(\Gamma(\alpha), \infty)$ can be extended to a holomorphic function with positive imaginary part in the upper half plane. It is the aim to show that this result can be generalized to cover a class of entire functions of genus 1 having all zeros on a half line.

Furthermore, similar results can be obtained for a class of entire functions of genus 2, including the so-called G-function of Barnes. The proofs use the theory of positive definite kernels and their relation to Nevanlinna-Pick functions.

The research is supported by The Danish Council for Independent Research — Natural Sciences.

David Sauzin, Professor, University of Pisa : *On the obtention of Ecalle-Voronin invariants via Resurgence theory*

Joint work with Artem Dudko.

For a holomorphic germ of the complex plane with a simple parabolic fixed point at the origin, we discuss the resurgent approach to the construction of the so-called "attracting and repelling Fatou coordinates" and the description of the "horn map" which classifies such germs up to analytic conjugacy. The method relies on the use of the Borel-Laplace summation for the formal iterator, which is the asymptotic expansion common to both Fatou coordinates, and then of Ecalle's alien operators. Our approach is influenced a lot by Ecalle's works of the 80s, but we tried to simplify the arguments and to use the language of holomorphic dynamics, in view of the current interest of many researchers in parabolic renormalization.

Luna Lomonaco, phd-student, University of Roskilde: *Parabolic-like mappings*

A polynomial-like mapping is a proper holomorphic map $f : U' \to U$, where $U', U \approx \mathbb{D}$, and $\overline{U'} \subset U$. This definition captures the behaviour of a poly- nomial in a neighbourhood of its filled Julia set. A polynomial-like map of degree d is determined up to holomorphic conjugacy by its internal and ex- ternal classes, that is, the (conjugacy classes of) the restrictions to the filled Julia set and its complement. In particular the external class is a degree d real-analytic orientation preserving and strictly expanding self-covering of the unit circle: the expansivity of such a circle map implies that all the periodic points are repelling, and in particular not parabolic. We extended the polynomial-like theory to a class of parabolic mappings which we called parabolic-like mappings. A parabolic-like mapping is thus similar to a polynomial-like mapping, but with a parabolic external class; that is to say, the external map has a parabolic fixed point, whence the domain is not contained in the codomain.

Organized by **Christian Berg, Christian Henriksen, Henrik Laurberg Pedersen, Carsten Lunde Petersen**