Holomorphic Day, April 4, 2008
Department of Mathematical Sciences
University of Copenhagen

Abstract

The purpose of the holomorphic day is to bring together people who use holomorphy in an essential way in their research. The event is supported by grant 272-07-0321 from FNU.

1 Schedule

The Lectures take place at the premises of the Department of Mathematical Sciences, Universitetsparken 5, Copenhagen.

Arrival, coffee, tea: 9.00-9.30 in E 419 (Fourth floor)

Bodil Branner: 9.30-10.15 in Aud. 9
Quasi-conformal mappings in holomorphic dynamics. The earliest occurences of quasi-conformal surgery.

Alexander Ulanovskii: 10.20-11.05 in Aud. 9
Completeness in $L^p(R)$ of discrete translates.

Henrik Laurberg Pedersen: 11.15-12.00 in Aud. 9
Properties of Gamma functions via complex methods.

Lunch: 12.00-13.15

Carsten Lunde Petersen: 13.15-14.00 in Aud. 7
Fatou-coordinates and -parameters in holomorphic dynamics -constructions and applications.

Eva Uhre: 14.05-14.50 in Aud. 7
Holomorphic dynamical systems – Structure of the space of quadratic rational maps.

Coffee break: 14.50-15.15
Anja Kabelka: 15.15-16.00 in Aud. 9
Dynamical limits of quadratic rational maps.

Christian Berg: 16.05-16.50 in Aud. 9
Iteration of the rational function $\psi(z) = z - 1/z$ and its relation to a Hausdorff moment sequence.

2 Abstracts

Bodil Branner, Technical University of Denmark: Quasi-conformal mappings in holomorphic dynamics. The earliest occurences of quasi-conformal surgery.

In holomorphic dynamics we study discrete dynamical systems given by iterations of a holomorphic mapping. The simplest interesting cases are complex quadratic polynomials. Any quadratic polynomial P is holomorphically conjugate to a unique one of the form $Q_c(z) = z^2 + c$. Using that the only holomorphic automorphisms of the complex plane are affine maps, this means that for a given P there exists an affine map $A(z) = az + b$ with $a \neq 0$ such that $A \circ P = Q_c \circ A$.

In 1981 Dennis Sullivan introduced quasi-conformal mappings as a tool in holomorphic dynamics. Allowing quasi-conformal conjugating maps he was able to conjugate certain quadratic polynomials of the form Q_c. From his work, completed by work of Adrien Douady and John H. Hubbard, it follows that any hyperbolic component of the Mandelbrot set is conformally isomorphic to the unit disc through an isomorphism, which is dynamically defined. Their constructions were the first examples of quasi-conformal surgery in holomorphic dynamics. The talk will illustrate the basic ideas behind.

Alexander Ulanovskii, University of Stavanger: Completeness in $L^p(R)$ of discrete translates.

A set of real numbers L is called a generating set for $L^p(R)$ if an L-generator exists, i.e. a function $f \in L^p(R)$ such that the set of all its L-translates $\{f(\cdot - l), l \in L\}$ spans $L^p(R)$. For $p = 1$, we show that L is generating for $L^1(R)$ if and only if the completeness radius of L is infinite. The main ingredient of the proof is to show that the L^1-generating sets are exactly the uiqueness sets forthe generalized Bernstein classes of entire functions. It is known that the set of integers is not generating for L^2 while each of its "small pertrubation" is.We study the effect of small perturbationsof integers on properties of corresponding generators. We show that there is a crucial difference between quasi-analytical and non quasi-analytical perturbations. In particular, our results extend "Landau's phenomenon" that certain perturbations of the trigonometrical system give exponential systems which are complete in L^2 on large sets. The proofs are based on some results on entire functions of exponential type due to de Branges and others.

The talk is based on joint results with J.Bruna and A.Olevskii.

Henrik Laurberg Pedersen, University of Copenhagen: Properties of Gamma functions via complex methods.

Recent results concerning Euler's gamma function and Barnes' double and triple gamma functions are obtained via complex methods, in particular the theory of Pick functions.

1. The remainder in asymptotic expansions of the logarithm of these gamma functions give rise to completely monotonic functions.

2. Some growth normalized functions related to the logarithm of Euler's and Barnes' functions are completely monotonic.

3. The median in the gamma distribution is a convex function of the parameter.

Some of these results are based on joint work with Christian Berg and with Stamatis Koumandos.

Carsten Lunde Petersen, Roskilde University: Fatou-coordinates and -parameters in holomorphic dynamics —constructions and applications.

The most interesting part of a family of holomorphic dynamical systems is the bifurcation locus. Parabolic maps, ie. with a parabolic periodic point are dense in the bifurcations locus and are rich sources of new dynamics. The main tool to understand the dynamics of parabolic maps are Fatou coordinates and Fatou parameters. Continuity properties of Fatou coordinates and parameters provides a way of linking the rich dynamical phenomena occuring near the parabolic map to the dynamics of the parabolic map. In this talk I will start by discussing ways of constructing such coordinates both for parabolic maps and for nearby maps. For dessert I will provide a few of the plentyful applications.

Eva Uhre, Roskilde University: Holomorphic dynamical systems – Structure of the space of quadratic rational maps.

The parameter space $\mathbf{poly}_2$ for the family of dynamical systems $(P_c)_{c\in\mathbb{C}}$, where $P_c : \mathbb{C} \to \mathbb{C}$ are the quadratic polynomials given by $P_c(z) = z^2+c$, is well understood. It is divided into the Mandelbrot set M, corresponding to maps with connected Julia sets, and its complement, corresponding to maps with totally disconnected Julia sets.

The space $\mathbf{poly}_2$ is naturally embedded as the "central slice" in the larger moduli space of all quadratic rational maps, called $\mathcal{M}_2$, where two maps belong to the same equivalence class if they are conjugate by some Möbius transformation. $\mathcal{M}_2$ is isomorphic to $\mathbb{C}^2$ and there is a subset, isomorphic to $\mathbb{D}\times\mathbb{C}$, of so-called polynomial–like maps wherein each slice is well understood as a quasi–conformal copy of $\mathbf{poly}_2$.

In the talk I will give a more elaborate description of the above mentioned properties and describe some results about the structure at the boundary of this subspace of polynomial–like maps, where we both retrieve phenomena from the polynomial–like case and see new phenomena emerging.

Anja Kabelka, Roskilde University: Dynamical limits of quadratic rational maps.

In this talk we will be concerned with sequences tending to infinity in the moduli space $\mathcal{M}_2$ of quadratic rational maps. For such a sequence the multipliers at the fixed points tend to μ, μ^{-1}, ∞, for some $\mu \in \hat{\mathbb{C}}$, and the maps suitably normalized converge to the degree 1 map $z \mapsto \mu z$. However if μ is a primitive q-th root of unity with $q > 1$, then the q-th iterate (suitably normalized) converges to a map of the form $G_T(z) = z + 1/z + T$.

For $|\lambda| < 1$ and c in the Mandelbrot set there is a unique quadratic rational map $R_{\lambda,c} \in \mathcal{M}_2$ with an attracting fixed point of multiplier λ that is hybrid equivalent to the quadratic polynomial $P_c(z) = z^2 + c$. If c is in the real 1/2-limb of the Mandelbrot set $[-2, -3/4]$ then $R_{\lambda,c}$ tends to infinity in $\mathcal{M}_2$ as λ tends to -1. We will discuss how the limiting map G_T of the second iterate $R^2_{\lambda,c}$ depends on c.

Furthermore we will determine all ideal limit points of the $\mathrm{Per}_m(0)$ curves, consisting of all conjugacy classes having a superattracting period m cycle.

Christian Berg, University of Copenhagen: Iteration of the rational function $\psi(z) = z - 1/z$ and its relation to a Hausdorff moment sequence.

There exists a uniquely determined function $f :]0, \infty[\to]0, \infty[$, satisfying the requirements

- $f(1) = 1$
- $\log(1/f)$ is convex
- $f(s) = \psi(f(s+1)), \quad s > 0.$

We prove that f has a meromorphic extension to the whole complex plane given by the iterative formula

$$f(z) = \lim_{n\to\infty} \psi^{\circ n}\left(\frac{1}{m_{n-1}}\left(\frac{m_{n-1}}{m_n}\right)^z\right),$$

where the numbers m_n are defined by

$$m_0 = 1, \quad m_{n+1} = \frac{-1+\sqrt{1+4m_n^2}}{2m_n}.$$

The numbers (m_n) form a Hausdorff moment sequence of a probability measure μ such that $\int t^{z-1}\,d\mu(t) = 1/f(z)$. The measure μ is a fixed point under the transformation $\widehat{T}$ of probability measures on $[0,1]$ given by

$$\int_0^1 \frac{1-t^{z+1}}{1-t}\,d\nu(t) \int_0^1 t^z\,d\widehat{T}(\nu)(t) = 1 \text{ for } \Re z \geq 0.$$

This is joint work with Antonio J. Durán, Sevilla.