[top]: X (and Y) topological, (H) and D l.s.c.
[con]: X (and Y) convex, $\forall y\ :\ \Phi(\cdot, y)$ concave,
[aff]: $\forall y\ :\ \Phi(\cdot, y)$ affine,
[dom] (domination): divergence dominates function of complete norm: $D(x, \widehat{\xi}) \geq f(\|x - \xi\|)$ with $f \geq 0$, $f(d_n) \to 0 \Leftrightarrow d_n \to 0$.

Ok!

Pinsker ineq: $D(P, \hat{Q}) \geq \frac{1}{2} V(P,Q)^2$

Under [aff], for a convex combination $\overline{x} = \sum \alpha_\nu x_\nu$,

$$\boxed{H(\sum \alpha_\nu x_\nu) = \sum \alpha_\nu H(x_\nu) + \sum \alpha_\nu\, \mathrm{D}(x_\nu, \widehat{\overline{x}})}$$

Cor.: H is concave

since: rhs = $\sum \alpha_\nu \Big(H(x_\nu) + \mathrm{D}(x_\nu, \widehat{\overline{x}}) \Big)$

$$= \sum \alpha_\nu \Phi(x_\nu, \widehat{\overline{x}}) = \Phi(\overline{x}, \widehat{\overline{x}}) = H(\overline{x})\,.$$

Add $\sum \alpha_\nu\, \mathrm{D}(x_\nu, y)$ to obtain *compensation identity*:

$$\boxed{\begin{aligned} \sum \alpha_\nu\, \mathrm{D}(x_\nu, y) &= \mathrm{D}(\sum \alpha_\nu x_\nu, y) + \sum \alpha_\nu\, \mathrm{D}(x_\nu, \widehat{\overline{x}}) \\ &= \sum \alpha_\nu\, \mathrm{D}(x_\nu, \widehat{\overline{x}}) + \mathrm{D}(\overline{x}, y)\,. \end{aligned}}$$

With [con] instead of [aff] only inequalities hold above.

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