Identification (1st case), avoiding Lagrange multipliers; exponential families

Standard setting: Φ, H, D ….

A strategy y^* is **robust** if $\Phi(x, y^*)$ is a constant independently of $x \in X$.

Theorem (*identification*). If (x^*, y^*) satisfy: $x^* \in X$, $y^* = \widehat{x^*}$, y^* robust (say $\forall x \in X : \Phi(x, y^*) = h$) then the game is in equilibrium and x^* and y^* are optimal strategies: $H(x^*) = H_{max}$, $R(y^*) = R_{min}$, $H_{max} = R_{min} = h$

<u>Proof</u>. $H(x^*) = \Phi(x^*, \widehat{x^*}) = \Phi(x^*, y^*) = h$

$R(y^*) = \sup_x \Phi(x, y^*) = h$

As $R(y^*) = H(x^*)$, result follows by minmax ineq. $\square$

<u>Key application</u>: Classical case. $\mathbb{A}$ **alphabet**, $E : \mathbb{A} \to \mathbb{R}$ **"energy function"**, $\bar{E} \in \mathbb{R}$ given **mean energy**, $X = \{P \in M_+^1(\mathbb{A}) \mid \langle E, P \rangle = \bar{E}\}$, the **preparation**.

Problem: Find MaxEnt distribution. Solution: Search among the **exponential family** $\mathcal{E}(X) = \{P \in M_+^1(\mathbb{A}) \mid \hat{P} \text{ robust}\}$

Clearly, any code κ of the form $\kappa = \alpha + \beta E$ is robust. As κ must be a code, $\sum e^{-\kappa_i} = 1$, hence

$\alpha = \ln Z(\beta)$ with $Z(\beta) = \sum e^{-\beta E_i}$ (Z the **partition function**).

So consider $(P_\beta, \kappa_\beta)_{\text{all } \beta \text{ s.t. } Z(\beta) < \infty}$ with $\kappa_\beta = \hat{P}_\beta$ and

$\kappa_\beta = \ln Z(\beta) + \beta E$ and choose β s.t. $P_\beta \in X$ - and you are done. $\square$