Identification (1st case), avoiding Lagrange multipliers
exponential families

Standard setting: $\Phi, H, D \ldots$
A strategy $y^*$ is robust if $\Phi(x, y^*)$ is a constant independently of $x \in X$.

**Theorem (identification).** If $(x^*, y^*)$ satisfy: $x^* \in X$, $y^* = \hat{x}^*$, $y^*$ robust (say $\forall x \in X: \Phi(x, y^*) = h$) then the game is in equilibrium and $x^*$ and $y^*$ are optimal strategies: $H(x^*) = H_{\text{max}}$, $R(y^*) = R_{\text{min}}$, $h_{\text{max}} = h_{\text{min}}$.

**Proof.**
$H(x^*) = \Phi(x^*, \hat{x}^*) = \Phi(x^*, y^*) = h$
$R(y^*) = \sup_x \Phi(x, y^*) = h$
As $R(y^*) = H(x^*)$, result follows by minmax ineq.

**Key application:** Classical case. NA alphabet, $E : \mathcal{A} \rightarrow \mathbb{R}$ "energy function", $\bar{E} \in \mathbb{R}$ given mean energy, $X = \{P \in M^*(\mathcal{A}) | \langle E, P \rangle = \bar{E} \}$, the preparation.

Problem: Find MaxEnt distribution. Solution: Search among the exponential family $\mathcal{E}(\mathcal{A}) = \{P \in M^*(\mathcal{A}) \mid \hat{P} \text{ robust} \}$

Clearly, any code $x$ of the form $x = \alpha + \beta E$ is robust.
As $x$ must be a code, $\sum E^{-x_i} = 1$, hence
$\alpha = \ln Z(\beta)$ with $Z(\beta) = \sum E^{-\beta E}$ (Z the partition function)

So consider $(P_\beta, x_\beta)_{\beta \in \mathbb{R}} \text{ s.t. } Z(\beta) < \infty$ with $x_\beta = \hat{P}_\beta$ and
$x_\beta = \ln Z(\beta) + \beta E$ and choose $\beta \text{ s.t. } P_\beta \in X$ - and you are done.