

Introduction to the Theory of  
Regular Exponential Families

by

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Misprints and changes

After the appearance of the lecture notes with the above title,  
some misprints and an error has been detected by Steffen Lauritzen  
and his students.

p.38 l. 1-4

should read

$$\alpha \leq P_{\delta}^V(K_V) = P_{\delta}^V(K_0), \quad P_{\delta}^V(K) \leq P_{\delta}^V(K_V) = P_{\delta}^V(K_0)$$

$$\text{and } \alpha = P_{\delta_0}^V(K_V) = P_{\delta_0}^V(K_0)$$

Integrating over  $v$  we find

$$\alpha \leq P_{\delta, \beta}(K_0), \quad P_{\delta, \beta}(K) \leq P_{\delta, \beta}(K_0)$$

$$\text{and } \alpha = P_{\delta_0, \beta}(K_0)$$

Which shows that  $K_0$  is unbiased and has larger power than  $K$ .

p.39 l. 9

homeomorphism

injective

p.40 l. 5<sup>-</sup>, 3<sup>-</sup>

int  $C$

0 twice

p.40 l. 6<sup>-</sup>

We shall assume

We shall assume a strong condition that even implies (7.2).

p.40 l. 5<sup>-</sup>

There exists

There exists a relatively open set  $0 \subset \text{int } C$  such that  $\tau(\theta(\beta)) \in 0, \beta \in I$ , and

p.41 l. 6, 8, 2<sup>-</sup>

int  $C$

0 three times

p.41 l. 9<sup>-</sup>

differentiable

differentiable at  $t = \tau(\theta(\beta))$

p.41 l. 8<sup>-</sup>

at  $t = \tau(\theta(\beta))$

erase

p.44 l. 8

$$\left(\frac{d\theta}{d\beta}\right)'$$

$$\left(\frac{d\theta}{d\beta}\right)'$$

$$\left(\frac{d\theta}{d\beta}\right)'$$

$$\left(\frac{d\theta}{d\beta}\right)'$$

p.46 l. 3<sup>-</sup>

$v_0$

$v_0 x$

p.46 l. 1<sup>-</sup>

$$\frac{d\theta}{d\alpha_0} j_0^{-1} \left(\frac{d\theta}{d\alpha_0}\right)' v_0$$

$$\frac{d\theta}{d\alpha_0} j_0^{-1} \left(\frac{d\theta}{d\alpha_0}\right)' v_0 \text{ where}$$

$j_0$  is the Fisher information about  $\alpha$  at  $\alpha = \alpha_0$ .

p.49 l. 11

$(0, j(\alpha_0))$

$(0, j^{-1}(\alpha_0))$

p.51 l. 6

$(0, j(\alpha_0))$

$(\alpha_0, \frac{1}{m} j^{-1}(\alpha_0))$ , where  $j(\alpha_0)$

is the information about  $\alpha$  at  $\alpha = \alpha_0$

p.52 l. 8 <sup>-</sup>	problem	problem
p.56 l. 2	differens	difference
p.62 l. 8 <sup>-</sup>	$(\hat{\tau} - \hat{\tau})$	$n(\hat{\tau} - \hat{\tau})$
p.71 l. 7	$U_1, \dots, U_k$	$U_i, U_j$
p.84 l. 9 <sup>-</sup>	$p^2$	$p^2$
p.87 l. 7 <sup>-</sup>	$x^3$	$f(x)$ twice
6 <sup>-</sup>	$dx$	$dx$ , and $f(x) = e^{x^3} - e^{-x^3}$ .
l. 3 <sup>-</sup>	$x^3$	$f(x)$
2 <sup>-</sup>	this	which
p.89 l. 3	Rothamsted Experi- mental Station	NAG Oxford
8	D.A.	J.A.
	Rothamsted Exponen- tial Station	NAG Oxford
p.91 l. 9	49-96	71-80

Correction of the proof of Lemma 1.7.

p.4 l. 7 a.s  $\mu$   $x_0, x \notin N_{\theta_0} \cup N_{\theta}$ , where  $N_{\theta}$  and  $N_{\theta_0}$  are null sets for  $\mu$ .

p.4 l. 8-14 This proof is incorrect. It should read as follows:

Since  $(\beta, u, v)$  is a minimal representation we can choose  $\theta_1, \dots, \theta_m$  such  $\beta(\theta_i) - \beta(\theta_0)$   $i = 1, \dots, m$  are linearly independent. We define  $B = (\beta(\theta_1) - \beta(\theta_0) : \dots : \beta(\theta_m) - \beta(\theta_0))$  and  $A = (\alpha(\theta_1) - \alpha(\theta_0) : \dots : \alpha(\theta_m) - \alpha(\theta_0))$ . Then for  $x \notin N_{\theta_0} \cup N_{\theta_1} \cup \dots \cup N_{\theta_m}$  we find  $B'(u(x) - u(x_0)) = A'(t(x) - t(x_0))$  which shows, for  $c = B'^{-1}A'$ , that  $u(x) - u(x_0) = c(t(x) - t(x_0))$  a.s.  $\mu$ .

Next we let  $V$  be the smallest vectorspace with the property that  $t(x) - t(x_0) \in V$  a.s. Then  $c : V \rightarrow \mathbb{R}^m$  is a surjection. Let  $u_1, \dots, u_m$  be linearly independent vectors in  $\mathbb{R}^m$  and let  $t_i$  have the property  $c(t_i) = u_i$ . Then  $t_1, \dots, t_m$  are linearly independent in  $V$ .

From the result just proved we find

$$(\alpha(\theta) - \alpha(\theta_0))' (t(x) - t(x_0)) = (\beta(\theta) - \beta(\theta_0))' c(t(x) - t(x_0))$$

a.s.  $u$  which shows that the functional

$$(\alpha(\theta) - \alpha(\theta_0))' - (\beta(\theta) - \beta(\theta_0))' c$$

vanishes on  $V$ , and that in particular for the vectors  $t_i$ , we get  $(\alpha(\theta) - \alpha(\theta_0))' t_i = (\beta(\theta) - \beta(\theta_0))' u_i$ .

Now let  $U = (u_1 : \dots : u_m)$ ,  $T = (t_1 : \dots : t_m)$  and  $b = U^{-1} T'$  then  $b(\alpha(\theta) - \alpha(\theta_0)) = \beta(\theta) - \beta(\theta_0)$ , which proves the first assertion. Finally we put  $\theta = \theta_i$  and get  $bA = B$  which shows  $bc' = I$ . Similarly  $cT = U$  shows that  $cb' = I$ .