# Niels Vigand Pedersen (1949—1996) :

# Some reflections

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**1.** I was deeply affected when Henrik Schlichtkrull of the University of Copenhagen, representing the organizing committee of this conference, wrote to me, suggesting that I make a few remarks about Niels Vigand Pedersen. I had known Niels only since 1988 but we had became very good friends and had much to talk about, not only in mathematics but about many other things. Although there are many here who knew him longer and more deeply than I had ever known him, nevertheless our paths intersected sufficiently often for me to gain a lasting impression of his personality. I wish to share some of these impressions with you.

Let me begin with a few biographical details[2]. Niels was born on March 12, 1949 and grew up in the village of Algestrup, not very far from Copenhagen. He started studying mathematics and physics at the University of Copenhagen when he was 19. His brilliance showed up almost at once and he obtained his master's degree in 1975, with a thesis written under the guidance of Esben Kehlet. He became interested in the theory of group representations and his life-long fascination with the representation theory of nilpotent and solvable groups began at this time, when he came across the remarkable papers of Dixmier, Kirillov, Moore, Auslander, Kostant, and Pukanszky. Eventually he himself was to make many

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[2] For these I am indebted to an obituary written by Professor Schlichtkrull.

beautiful contributions to this subject, and by the time of his death was regarded as one of the leading experts in this area. In fact, at the time of his death he had discovered many new themes and results. His passing away when he was at the peak of his creative powers is a real tragedy for his mathematical colleagues and for mathematics in Denmark in general, but above all, an irreconcilable loss to his family.

After his master's degree his involvement with the representation theory of nilpotent and solvable Lie groups became deeper. Visits to the Universities of Pennsylvania and Berkeley were critical in his maturation and development, especially contacts with Pukanszky and Michele Vergne. His interaction with Pukanszky was especially deep and fruitful. They became good friends and remained so as long as they lived. They were in fact in the middle of some correspondence about writing a book together on representation theory when Pukanszky died. I think the loss of a mentor and a teacher of exceptional stature like Pukanszky affected Niels deeply, especially as he himself was very ill at that time.

Niels joined the University of Copenhagen as a lektor (associate professor) in 1986, and remained there till the end. He was a very inspiring lecturer, and his interest in physics and other sciences allowed him to reach out to a very wide circle of students. In recognition of his ability as well as his dedication to teaching he was given the teaching award of the Science Faculty in 1994.

I shall discuss his scientific work a little later in more detail, but let me mention here that he had great interest in computer algebra, especially in the development of algorithms at a very sophisticated level. He really understood the intimate relationship between proofs and their implementability by algorithms. Actually, in a case that I knew personally, the computerized calculations led him to a beautiful conjecture that he then was able to prove, to his great satisfaction. He represented Denmark in Euromath for some time, and I am sure his knowledge and enthusiasm for the role of computers in modern mathematics was a source of great help and inspiration to his colleagues in that organization.

He became the Chair of the mathematics department at the University of Copenhagen in 1993. It was not a particularly easy time to be at the helm since the department (like many other departments all over the world) was struggling to reach a consensus on a number of difficult issues. To try to resolve some of them an Advisory Group was created whose task was to undertake a review of the work of the departments of mathematics, statistics, and actuarial science. Niels invited me to be a member of this group. I visited Copenhagen several times during the next two years. The task before our group was a complicated one, and the principal reason why we were able to come up with a unanimous report that was substantially acceptable to the departments concerned was the collaboration of Niels. He was the epitome of courtesy and consideration for us, but at the same time represented the interests of the department with great decisiveness and integrity. I became very fond of him during this period, and became one of his admirers, not only for his abilities in mathematics and administration, but also because of his charm, sense of humour, and total lack of pretension.

Then, suddenly, in the autumn of 1995, he was diagnosed as having a malignant brain tumour. The tumour was unexpectedly and uncontrollably aggressive, and surgery could not succeed in stopping its spread. He continued giving lectures almost till the end but had to stop as the illness became too severe. The end came on November 24, 1996 and I learnt of it from Professor Schlichtkrull almost immediately after. He is mourned by his friends and colleagues, and above all by his wife Maria and their three children, Andreas, Kathrine, and Louise.

**2.** Niels's work is in the area of representations of Lie groups, especially nilpotent and solvable groups. This theory, for which Mackey's work had provided the foundations, is the creation of Dixmier and Kirillov in the nilpotent case, and of Moore, Auslander-Kostant, Pukanszky, and many others in the solvable case. The central themes were well understood by the time Niels started his career, and it is a tribute to his penetration and persistence that he was able to obtain many beautiful results in this mature topic. Starting from around 1988 he discovered a more direct approach to many aspects of this theory that did not rely on induction but was based on ideas from quantization and symplectic geometry. In what follows I shall attempt, in a limited way, to give you a feeling for his ideas. He carried out his program in the papers [P1]—[P5] and had obtained results for many additional papers when his life was cut short. I think these papers reveal him at his characteristic best.

One of the guiding principles in the representation theory of Lie groups is that there is—or there should be—a correspondence between the representations of a Lie group and the orbits of the action of the Lie group on the dual of its Lie algebra. Actually this is one of the most fertile and profound principles that governs all of representation theory; not only for solvable groups but even for semisimple groups. It relates representation theory to symplectic geometry, dynamical systems, and quantization, which are themes of unsurpassed beauty and scope. To mention only one aspect of this correspondence, the orbits, being symplectic manifolds, allow one to do Hamiltonian mechanics on them, and the unitary representation associated to an orbit may therefore be regarded as the quantization of the kinematics of this Hamiltonian manifold. Thus, in some way, the structure and geometry of the orbits gives one a brief glimpse into the much richer world of irreducible unitary representations of the group.

Let $G$ be a Lie group, connected and simply connected, and $\mathfrak{g}$ its Lie algebra. Then $G$ has a natural action on $\mathfrak{g}$ through its adjoint representation $x \mapsto Ad(x)$, and hence, a natural action on $\mathfrak{g}^*$, the dual of $\mathfrak{g}$. We write $(x, X) \mapsto xX$ and $(x, \ell) \mapsto x\ell$ for these actions. For any $\ell \in \mathfrak{g}^*$, let $G_\ell$ be its stabilizer in $G$ and let $\mathfrak{g}_\ell$ be the Lie algebra of $G_\ell$. The orbit $O_\ell \simeq G/G_\ell$ of $\ell$ is then a regularly imbedded submanifold and carries a natural $G$-invariant symplectic structure. In fact, the bilinear form

$$X, Y \mapsto \ell([X, Y]) \quad (X, Y \in \mathfrak{g})$$

has $\mathfrak{g}_\ell$ for its radical and so induces a symplectic form on $\mathfrak{g}/\mathfrak{g}_\ell \times \mathfrak{g}/\mathfrak{g}_\ell$, which can be identified with $T_\ell(O_\ell) \times T_\ell(O_\ell)$; here $T_\ell(O_\ell)$ is the tangent space to $O_\ell$ at $\ell$. This form can then be transported to all the points of the orbit by the action of $G$, and the resulting 2-form $\omega_\ell$ is smooth and closed. One can then in various ways construct a line bundle on a suitable quotient space of $O_\ell$ on which $G$ acts, and the resulting action of $G$ on the space of sections of this line bundle is a representation of $G$ that is associated to the orbit $O_\ell$. The theories of Kirillov, Auslander-Kostant, Pukanszky, and others all have the goal of describing these line bundles precisely and showing that all irreducible unitary representations of the group $G$ may be obtained by this procedure, when $G$ is nilpotent, or more generally, solvable. This procedure, with its many generalizations and variants, is known as *geometric quantization*. In fact, if $G$ is the three dimensional Heisenberg group, it leads to the representation that is central to (one dimensional) quantum mechanics, involving the famous operators $q, p$ and the commutation rule

$$pq - qp = \frac{\hbar}{i} I$$

The Kirillov theory, which dealt with nilpotent groups, and which served as the model for all subsequent work along this path, is especially simple and beautiful. Given $\ell$, one chooses a maximal isotropic subspace of $\mathfrak{g}$ for the form $\omega_\ell$ that is also a subalgebra, say $\mathfrak{h}_\ell$; if $H_\ell$ is the analytic subgroup of $G$ defined by $\mathfrak{h}_\ell$, then $H_\ell$ is closed and carries a unitary character (=one dimensional representation) $\chi_\ell$ such that

$$\chi_\ell(\exp X) = e^{i\ell(X)} \quad (X \in \mathfrak{h}_\ell).$$

The unitary representation of $G$ obtained by inducing $\chi_\ell$ from $H_\ell$ is then irreducible, and the class of this representation depends only on the orbit $O_\ell$, and not on the choice of $\ell$ within the orbit or the choice of $\mathfrak{h}_\ell$. All irreducible unitary representations of $G$ are obtained this way. With suitable modifications, some of which require new ideas going beyond the Kirillov theory, this scheme can be extended to all solvable $G$; the extension is particularly nice when $G$ is in addition *exponential*, namely when the exponential map is a diffeomorphism from $\mathfrak{g}$ to $G$. Then $G_\ell$ is connected and the above description extends with few changes (for an excellent description of these basic facts, see [M]).

Now $O = O_\ell$ is foliated by the orbits $H_\ell{\cdot}\ell$, and the quotient space of the orbit by this foliation is the space $M = G/H_f$ where $f$ is a fixed point of $O_\ell$. $M$ is the *configuration space*, the functions on $M$ define the *position observables*, and the functions that are linear on the fibres of the map

$$O_\ell \longrightarrow M, \qquad sf \longrightarrow sH_f$$

are the *momenta.*

In this context it is of course natural to ask whether one can construct *global coordinates*

$$q_1, q_2, \ldots, q_{d/2}, p_1, p_2, \ldots, p_{d/2} \qquad (d = \dim(O))$$

on $O$ so that the symplectic form reduces to

$$dp_1 \wedge dq_1 + \ldots + dp_{d/2} \wedge dq_{d/2}$$

Such coordinates are called *canonical* and they always exist locally; this is the statement of the classical theorem of Darboux. But enlarging a canonical chart globally or even semiglobally is another matter. That this can be done for nilpotent groups was known to be true [ACMP] [V] when Niels got interested in this problem. He wanted to develop a computer algorithm for writing down the canonical coordinates for all nilpotent groups and all coadjoint orbits. But, as he told me when I was visiting Copenhagen in 1988, he noticed that the algorithm also applied to many solvable groups and supplied *global canonical coordinates* on the coadjoint orbits [P1]. He was then able to prove a very general theorem from which the existence of the canonical coordinates followed for all coadjoint orbits of all exponential solvable groups [P2]. This is a perfect illustration of how his mind operated; there was total harmony between the computational and conceptual aspects of mathematics in his mind. Recently, together with a student of mine, I have been looking into the Lie theoretic aspects of the dynamics of Riemann (=self-gravitating) ellipsoids and ellipses, and it turns out that for the case of the rotating ellipses (which for instance model rotating galaxies), although the Lie group in question is not solvable, nevertheless there is a reduction to the solvable case and by the theorem of Niels one can explicitly construct the canonical coordinates.

As I have mentioned above, Niels deduced the existence of global canonical coordinates from a more general theorem which he was able to establish. He considers a connected $G$ not necessarily solvable, and a coadjoint orbit $O$ for $G$. He makes the assumption that there is a *real polarization* on this orbit that is covariant with respect to $G$, defined by a maximal isotropic subalgebra $\mathfrak{h}$ of $\mathfrak{g}$, and makes a deep analysis of the sheaf of Lie modules of *quantizable functions* on $O$. To simplify matters I shall assume that the stabilizers of points of $O$ are connected and that the leaves of the polarization are flat affine subspaces of $\mathfrak{g}$; in the absence of these conditions the arguments become more technical but no new conceptual aspect is involved. For exponential solvable groups matters can be arranged so that these conditions are always satisfied. Thus $f$ is a fixed point of $O$, $G_f$, the stabilizer of $f$ in $G$, is connected with Lie algebra $\mathfrak{g}_f$, and $\mathfrak{h}$ is a subalgebra of $\mathfrak{g}$ that is maximal isotropic for the symplectic form $X, Y \mapsto f([X, Y])$. Clearly

$$\mathfrak{g}_f \subset \mathfrak{h}$$

The analytic subgroup $H$ defined by $\mathfrak{h}$ is known to be always closed, and the quotient of $O$ by the foliation defined by the polarization is a manifold $M$ identified with $G/H$. The condition mentioned above on the flatness of the leaves is equivalent to the condition that

$$Hf = f + \mathfrak{h}^{\text{ann}}$$

where $\mathfrak{h}^{\text{ann}}$ is the annihilator of $\mathfrak{h}$. This is known as the *Pukanszky condition.* It is known that if $G$ is exponential solvable, and $f \in \mathfrak{g}^*$ is arbitrary, there is always a real polarization $\mathfrak{h}$ that satisfies the Pukanszky condition. The smooth locally defined functions on $M$, also viewed as smooth locally defined functions on $O$ via the natural map $\pi : O \longrightarrow M$, form a sheaf $\mathcal{Q}$ on $M$, the *sheaf of the position observables*; for any two local sections $q_1, q_2$ of this sheaf, we have

$$\{q_1 \circ \pi, q_2 \circ \pi\} = 0$$

To introduce the momenta we have to consider the larger sheaf $\mathcal{P}$ on $M$ with the property that for any open set $U \subset M$, $\mathcal{P}(U)$ is the space of smooth functions $p$ on $\pi^{-1}(U)$ that normalize the space of position observables on $\pi^{-1}(U)$ (with respect to $\{\ ,\ \}$, i.e., for each $q \in \mathcal{Q}(U)$

$$\{p, q \circ \pi\} = q' \circ \pi \text{ for some } q' \in \mathcal{Q}(U) \qquad (U \text{ open in } M)$$

Then

$$U \longmapsto \mathcal{P}(U)$$

is a sheaf of Lie algebras and at the same time a sheaf of modules for the sheaf of functions $\mathcal{Q}$. We call the pair $(\mathcal{P}, \mathcal{Q})$ the *quantizable sheaf of Lie modules on $M$*. On the other hand, each $X \in \mathfrak{g}$ may be viewed as a linear function on $\mathfrak{g}^*$, and the restriction of this function to $O$ is denoted by $\psi^X$; thus $\psi^X(\ell) = \langle X, \ell \rangle$. It is an easy exercise to show that these functions are global sections of the sheaf $\mathcal{P}$. Niels shows that $\mathcal{P}$ is generated as a module by the $\psi^X (X \in \mathfrak{g})$ over $\mathcal{Q}$, and that $\mathcal{P}$, as a sheaf of modules over $\mathcal{Q}$, is *locally free.*There is another sheaf of Lie modules over $M$, in fact an infinity of them, defined as follows. Consider the character (not necessarily unitary)

$$\chi : \exp X \longmapsto e^{\kappa f(X)} \qquad (\kappa \in \mathbf{C})$$

Then $\chi$ gives rise to a homogeneous line bundle $L_\chi$ on $M$, and one has the sheaf of differential operators $\mathcal{D}_\chi$ of order $\leq 1$ operating on the sections of $L_\chi$, which is also a sheaf of Lie modules over $\mathcal{Q}$. Niels's fundamental theorem [P2] asserts that the two sheaves $\mathcal{P}$ and $\mathcal{D}_\chi$ are naturally isomorphic as Lie modules, i.e., the isomorphism preserves the Lie algebra structure as well as the module structure of the sheaves. If now we assume that $M$ has a global chart consisting of the coordinates $t_u$ (this is the case when $G$ is exponential solvable), Niels's theorem shows that there are functions $q_u, p_u$ on $O$ that correspond to $t_u, \partial/\partial t_u$ under his isomorphism. It is then immediate that the $q_u, p_u$ are canonical coordinates for $O$. The isomorphism he constructs is sufficiently transparent that an algorithm can be developed for getting the $q_u, p_u$, which is what he did in [P1] and [P3].

Niels did not stop with this result. He wanted to know how the canonical coordinates he constructed changed when the orbit was changed;

he also wanted to explore the relationship between his isomorphism and the unitary representation of $G$ that one can construct on the space of sections of the line bundle $L_\chi$ when $\chi$ is unitary, i.e., $\kappa$ is pure imaginary. The first question is answered in [P3] in the case when the ambient group $G$ is a connected, simply connected nilpotent Lie group. Then the group and all its coadjoint orbits have *polynomial structures* derived ultimately from the polynomial structure on the Lie algebra $\mathfrak{g}$ that results from identifying $G$ with $\mathfrak{g}$ and viewing multiplication $G \times G \longrightarrow G$ as a map $\mathfrak{g} \times \mathfrak{g} \longrightarrow \mathfrak{g}$. It then makes sense to ask whether a polynomial version of the main theorem of [P2] can be proved. The paper [P3] carries out this program. A consequence of this is the derivation of an earlier result of Vergne [V] that one can find rational functions $q_1, q_2, \ldots, q_e, p_1, p_2, \ldots, p_e$ that restrict on each orbit belonging to a specified stratum to canonical coordinates on that orbit. Needless to say that Niels's methods are algorithmic, and he indicates in [P3] how these algorithms can be made very explicit.

The second question mentioned above is even more interesting. Its study led Niels to a direct construction of the irreducible representation associated to the orbit. Let $G$ be a connected simply connected exponential solvable Lie group and let other notation be the same. We choose $\kappa = i$ and assume that $\mathfrak{h}$ is a real polarization satisfying the Pukanszky condition. The canonical coordinates on $M$ are

$$q_1, q_2, \ldots, q_{d/2}, p_1, p_2, \ldots, p_{d/2}$$

The functions in $\mathcal{P}(U)$ are precisely those that can be expressed(uniquely) in the form

$$\psi = \sum_{u=1}^{d/2} a_u p_u + a_0$$

where $a_0, a_1, \ldots, a_{d/2} \in C^\infty(U)$. In particular, for any $X \in \mathfrak{g}$ we can write the global expression

$$\psi^X = \sum_{u=1}^{d/2} a_{X,u} p_u + a_{X,0} \qquad (a_{X,u}, a_{X,0} \in C^\infty(M))$$

Identifying $M$ with $\mathbf{R}^{d/2}$ via the coordinates $q_u$, Niels obtains the following theorem : there exists a unique unitary representation $\pi$ of $G$ on $\mathcal{H} =$

$L^2(\mathbf{R}^{d/2})$ such that $C_c^\infty(\mathbf{R}^{d/2}) \subset \mathcal{H}_\infty$ [3], and such that for all $X \in \mathfrak{g}$ and all $f \in C_c^\infty(\mathbf{R}^{d/2})$,

$$d\pi(X)f = \sum_{u=1}^{d/2} a_{X,u} \frac{\partial f}{\partial t_u} + i a_{X,0} f + \frac{1}{2} \sum_{u=1}^{d/2} \frac{\partial a_{X,u}}{\partial t_u} f$$

Niels was able to show, without using induction, that in the nilpotent case the image of the universal enveloping algebra under $d\pi$ is precisely the *Weyl algebra* of polynomial coefficient differential operators on $\mathbf{R}^{d/2}$.

The paper [P4] is a change of direction in his work. In it Niels studies matrix coefficients and quantization for nilpotent groups. I would like to venture the suggestion that he wanted to develop a complete *operator Fourier transform theory* on nilpotent groups and the results of [4] were to be the foundation for such a theory. If $\pi$ is an irreducible unitary representation of a connected simply connected nilpotent group $G$, then for any function $\varphi$ in the Schwartz space of $G$ (defined by carrying over the Schwartz space of $\mathfrak{g}$ via the exponential map), $\pi(\varphi)$ is a smooth operator[4] and the map $\varphi \mapsto \pi(\varphi)$ is surjective. But this is far from injective and for an operator transform theory one has to somehow "invert"this map. By deep Fourier analysis Niels achieves this. He constructs a subspace $\mathfrak{g}_e$ of $\mathfrak{g}$ where $e$ is a set of discrete parameters of the orbit corresponding to $\pi$, and shows that the map $\varphi \mapsto \pi(\varphi)$ factors through the restriction $\varphi \circ \exp\big|_{\mathfrak{g}_e}$ inducing an isomorphism of the Schwartz space $\mathcal{S}(\mathfrak{g}_e)$ with the nuclear space of smooth operators associated to $\pi$, and gets an explicit formula for the inverse of this map. He then sets up an isomorphism between the Schwartz space $\mathcal{S}(O)$ of the orbit $O$ associated to $\pi$ and $\mathcal{S}(\mathfrak{g}_e)$. In fact, for $a \in \mathcal{S}(O)$, define $\check{a}$ as a function on $\mathfrak{g}$ by

$$\check{a}(X) = \int_O a(\ell) e^{-i\langle X,\ell\rangle} d\beta_O(\ell) \qquad (X \in \mathfrak{g})$$

where $\beta_O$ is the canonical (suitably normalized) invariant measure on $O$. Then $\check{a}$ is in $C^\infty(\mathfrak{g})$, and its restriction to $\mathfrak{g}_e$ is in $\mathcal{S}(\mathfrak{g}_e)$. The map

$$a \longmapsto \check{a}\big|_{\mathfrak{g}_e}$$

[3] The subscript $\infty$ refers to the subspace of smooth vectors.

[4] A bounded operator $A$ in the space of $\pi$ is smooth if $d\pi(u)A$ and $Ad\pi(u)$ are bounded operators for all elements $u$ of the universal enveloping algebra of $\mathfrak{g}$.

is the topological linear isomorphism between $\mathcal{S}(O)$ and $\mathcal{S}(\mathfrak{g}_e)$ described above. Niels uses this isomorphism to set up a far reaching generalization of the Weyl correspondence (in quantum mechanics) taking functions in $\mathcal{S}(O)$ to smooth operators in the space of $\pi$. This correspondence extends to the duals of these spaces, and reduces to the Weyl correspondence when the group is the Heisenberg group. The assignment

$$a \longmapsto Op_\pi(a)$$

that he constructs is a topological linear isomorphism of $\mathcal{S}(O)$ with the nuclear space of smooth operators in the space of $\pi$ and has many elegant properties. Among these are

$$\begin{aligned} Op_\pi(a)^* &= Op_\pi(\bar{a}) \\ \text{Tr } (Op_\pi(a)) &= \int_O a(\ell) d\beta_O(\ell) \\ \text{Tr } (Op_\pi(a) Op_\pi(b)) &= \int_O a(\ell) b(\ell) d\beta_O(\ell) \end{aligned}$$

He showed that this isomorphism gives rise to an isomorphism between the dual spaces; denoting it again by $Op_\pi$, he showed that the image of the polynomial functions on $O$ is the image of the enveloping algebra under $d\pi$; in particular

$$Op_\pi(\psi^X) = d\pi(X)$$

He mentions in the introduction to this paper that he had obtained extensions of many of these results to the case when $G$ is solvable.

Niels himself regarded this paper as a breakthrough and had succeeded in generalizing many of its results to solvable groups at the time of his death. I think this paper represents Niels at his most imaginative, and makes us wonder what he might have done in this direction had he lived longer.

I have touched only a few small aspects of his work. In addition to his published work there were a number of additional results that he had mentioned from time to time, to me and to some of his other friends and colleagues. For instance, it appears that he had developed a computer algorithm for constructing, for any Lie algebra of finite dimension, a faithful finite dimensional linear representation of it. Information on downloading the programs and examples of solvable Lie groups will be available at

http://www.math.purdue.edu/~rcp/pedersen.html

**3.** I would now like to be allowed to become a little more personal. As I mentioned earlier I first met Niels in 1988 when I was visiting the University of Copenhagen for a quarter. We started exchanging reprints and preprints, but our real friendship began in 1993 when he invited me to be a member of the Advisory group that was to conduct a review of the departments of mathematics, statistics, and actuarial science. I visited Copenhagen many times and enjoyed the many social occasions we were together. I remember a trip to Malmö by just the two of us, which he arranged at my request. He was disappointed that we had to go in an ordinary boat and not in a hydrofoil as I wanted (I had always wanted to go on one of these after reading the novels about Martin Beck, the famous detective created by Maj Sjöwall and Per Wahlöö). It was a delightful visit, and we wandered through the downtown area. Niels was in a very good mood, and we were constantly laughing at his humorous comments on and imitations of the oddities and eccentricities of Scandinavians. I remember it was very cold even though winter was officially over, and we had to take refuge inside a pub long before the accepted dinner time so that by the time we were ready for dinner we were also very drunk.

The meetings with students and other faculty in connection with the Advisory committee that I had mentioned earlier were arranged by him. He participated in them fully, and his impartiality led to some very lively exchanges and made my visit memorable to me. Of the greatest importance for the success of our efforts was his view of mathematics that emphasized the close interplay of mathematics and the physical sciences and its role in understanding the world around us. We always ended the visits with a dinner at a very nice downtown restaurant. At one of these dinners he described his development as a mathematician and the impact that Pukanszky had on him. I told him, and he agreed with me, that it was a fortunate event that he and Pukanszky found each other. I think Niels was the one person that fully satisfied the exalted expectations that Pukanszky had for his ideal disciple. I visited his home during one of these visits and had the chance to meet him at home, with his family and a few close friends, and understood what a harmonious life he had built up.

On November 25, 1995 I received a devastating message from him, telling me that he had what was perhaps an inoperable brain tumour, and that he was preparing to die. I tried to reach out to him and to lift his spirits. The months that followed were very discouraging for me personally. I tried to console him as much as I could, but more and more I myself did not seem to believe the words that came from me. We corresponded as much as his energies would allow him, and gradually, through the surgery and its extremely difficult aftermath, it became clear to him that he was fighting a losing battle. His letters to me expressed his despair but also revealed his gradual emergence from his depression and his eventual acceptance of what was happening to him, as well as his will to live his remaining days doing what he had always liked to do—lecture, read and think about mathematics. He could still laugh, even during those dark days, at some of the stories about Pukanszky. His last letter to me was on October 1, 1996. He was sad that he had to stop lecturing about a week earlier. But I could sense that he had attained an inner tranquillity that allowed him to look at himself with detachment; his real sorrow was not only for his own situation but also for what it meant to those he was leaving behind. His was a lofty and courageous soul, and I cannot describe my feelings any better than by quoting from a letter that Beverly Kleiman wrote to me : "Niels was special and those who were lucky enough to have him as a friend know what they have lost."

**4.** I suppose the manner in which we reconcile ourselves to the unexpected and untimely death of people close to us depends very much on our cultural background. I would like to say a few words about this as someone who grew up in an ancient land with an ancient culture. I have frequently reflected on the tale of *Naciketas*, especially during times of great distress. *Naciketas* was a very young boy in an ancient epoch. His father was one day performing a sacred ritual that involved worship of the gods and the giving away of property and wealth in charity. Naciketas, while watching the ritual his father was performing, noticed that whenever something had to be given away, his father always chose to get rid of things that he no longer needed or could make no use of. The boy felt this was utterly shameful and repeatedly asked his father to whom he (Naciketas) himself was going to be given away. His father was of course thoroughly irritated by this time, and in a moment of anger told Naciketas that he was to be given away to *Yama,* the Lord of death. Of course, once made, the promise could not be recalled, and so Naciketas went to the abode of Yama. The Lord of death was surprised by Naciketas's untimely appearance and found out from the child what had happened. He was astounded by the wisdom and courage shown by Naciketas and told him that he would be able to go back to his father, that his father would bear no animus against him, and offered further that he would grant anything that the boy asked for. To Yama's surprise Naciketas asked to be instructed on the mystery of life and death. Yama was reluctant to do this, but Naciketas held firm, refusing to accept many wonderful things that Yama offered, while insisting that there was no one more qualified to explain to him the mystery of life and death than the Lord of death himself. So finally Yama yielded and proceeded to explain the meaning of life to Naciketas. The essence of Yama's teaching is that all lives must end, that the journey through one's life is what prepares us for our end, and that the ideal way to make this journey is to steadfastly pursue deeper knowledge and harmony. These remarks of Yama and his dialogue with Naciketas form the content of one of the most famous of all ancient Hindu Scriptures, the *Katha Upanishad.* Viewed from this perspective, the journey of Niels Vigand Pedersen was a perfect one.

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