GEOM2, 2010

P9. Class program for Friday Dec 18

Select from the following exercises

1 Prove Lemmas 5.1.1 and 5.1.2

2 Let X, Y be a Hausdorff topological spaces, and let $K \subset X$ be compact.

a) Prove that $K \times \{y\}$ is compact for each $y \in Y$.

b) Prove that if $K \times \{y\} \subset W$ for some open set W in $X \times Y$, then there exist open sets $U \subset X$ and $V \subset Y$ such that $K \times \{y\} \subset U \times V \subset W$.

3 Let X, Y be a Hausdorff topological spaces, and let $K \subset X$, $L \subset Y$ be compact. Prove that $K \times L$ is compact.

(Hint: If an open cover of $K \times L$ is given, apply (a) above to obtain, for each y, a finite subcover of $K \times \{y\}$. Then apply (b) above to show that finitely many of these subcovers will cover the entire set $K \times L$.)

4 Let M and N be abstract manifolds, and let $(f_\alpha)_{\alpha \in A}$ and $(g_\beta)_{\beta \in B}$ be partitions of unity of them. Show that the collection of functions on $M \times N$ given by all products $f_\alpha(x) g_\beta(y)$ is a partition of unity on $M \times N$.