GEOM2, 2010

P7. Class program for Friday Dec 11

Select from the following exercises

1 Verify the statement below Example 4.2.4, about transitivity of the property of being a submanifold.

2 Let M, N be abstract manifolds and fix an element $y_0 \in N$. Prove that the subset $\{(x, y_0) \mid x \in M\}$ is a submanifold of $M \times N$.

3 Let M be an abstract manifold. Prove that the diagonal $\{(x, x) \mid x \in M\}$ is a submanifold of $M \times M$.

4 Let L, M be abstract manifolds, and let $N \subset M$ be a submanifold. Prove that a map $f\colon N \to L$ is smooth if and only if, for each $p \in N$ there exists an open neighborhood W of p in M, and a smooth map $F\colon W \to L$ such that the functions f and F agree on $W \cap N$.

5 Verify that the image by π of the equator of S^2 is a submanifold in $\mathbb{R}P^2$, which is diffeomorphic to $\mathbb{R}P^1$ and to S^1 (see also Exercise P3-7).

6 Prove the following converse to Theorem 4.4. Let L be an l-dimensional submanifold of an m-dimensional abstract manifold M. Prove that for each $p \in L$ there exists an open neighborhood W of p in M, a smooth map $f\colon W \to \mathbb{R}^{m-l}$ such that $y = f(p) \in \mathbb{R}^{m-l}$ is a regular value and $L \cap W = f^{-1}(y)$ (hint: Apply Theorem 4.3).