P7. Class program for Friday Dec 11

Select from the following exercises

1 Verify the statement below Example 4.2.4, about transitivity of the property of being a submanifold.

2 Let $M, N$ be abstract manifolds and fix an element $y_0 \in N$. Prove that the subset $\{(x, y_0) \mid x \in M\}$ is a submanifold of $M \times N$.

3 Let $M$ be an abstract manifold. Prove that the diagonal $\{(x, x) \mid x \in M\}$ is a submanifold of $M \times M$.

4 Let $L, M$ be abstract manifolds, and let $N \subset M$ be a submanifold. Prove that a map $f: N \to L$ is smooth if and only if, for each $p \in N$ there exists an open neighborhood $W$ of $p$ in $M$, and a smooth map $F: W \to L$ such that the functions $f$ and $F$ agree on $W \cap N$.

5 Verify that the image by $\pi$ of the equator of $S^2$ is a submanifold in $\mathbb{R}P^2$, which is diffeomorphic to $\mathbb{R}P^1$ and to $S^1$ (see also Exercise P3-7).

6 Prove the following converse to Theorem 4.4. Let $L$ be an $l$-dimensional submanifold of an $m$-dimensional abstract manifold $M$. Prove that for each $p \in L$ there exists an open neighborhood $W$ of $p$ in $M$, a smooth map $f: W \to \mathbb{R}^{m-l}$ such that $y = f(p) \in \mathbb{R}^{m-l}$ is a regular value and $L \cap W = f^{-1}(y)$ (hint: Apply Theorem 4.3).