GEOM2, 2010

P4. Class program for Tuesday Nov 30

Presentation of E1. Besides select from the following exercises

1 Let $U \subset \mathbb{R}^m$ be an open set, and let $h_1\colon U \to \mathbb{R}^{k_1}$ and $h_2\colon U \to \mathbb{R}^{k_2}$ be smooth maps. Let $\mathcal{S}_1 \subset \mathbb{R}^{m+k_1}$ and $\mathcal{S}_1 \subset \mathbb{R}^{m+k_2}$ denote the graphs of the two maps. Prove that the map $(x, h_1(x)) \mapsto (x, h_2(x))$ is a diffeomorphism of $\mathcal{S}_1$ onto $\mathcal{S}_2$.

2 Let S^2 denote the unit sphere in $\mathbb{R}^3$, and define a map $f\colon S^2 \to S^2$ by

$$f(x, y, z) = (x\cos(z) + y\sin(z), x\sin(z) - y\cos(z), z)$$

Prove that f is a diffeomorphism.

3 Let $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ be given by

$$F(x, y, z) = (xy, yz, zx),$$

and define $f\colon \mathbb{R}P^2 \to \mathbb{R}^3$ by $f(\pm(x, y, z)) = F(x, y, z)$ for $(x, y, z) \in S^2$. Prove that f is smooth, but not injective.

4 Verify that the set of 2×2 upper triangular matrices with determinant $\neq 0$ is a Lie group (with matrix multiplication as the product).

5 Let $f_1\colon M_1 \to \tilde{M}_1$ and $f_2\colon M_2 \to \tilde{M}_2$ be smooth maps. Verify that $f_1 \times f_2$ is smooth $M_1 \times M_2 \to \tilde{M}_1 \times \tilde{M}_2$

Let G_1, G_2 be Lie groups. Verify that the product $G_1 \times G_2$ is a Lie group.