GEOM2, 2010-2011

P11. Class program for Tuesday Jan 11

Presentation of E5. Besides select from the following exercises

1 Let Y denote the vector field $Y(x, y) = (x, -y)$ on $\mathbb{R}^2$. Determine its coordinates with respect to the chart $\sigma(u, v) = (u + v, u - v)$.

2 Prove that $Y(x, y, z) = (xz, yz, z^2 - 1)$ is a smooth vector field on the sphere S^2.

3 Consider $M = \mathbb{R}^2$. Compute the Lie bracket $[X, Y]$ when
a) $X(x, y) = (x, 0), Y(x, y) = (0, y)$
b) $X(x, y) = (0, x), Y(x, y) = (y, 0)$.

4 Consider $M = \mathbb{R}$. Determine the Lie bracket of two arbitrary smooth vector fields $X(x) = f(x)$ and $Y(x) = g(x)$.

5 Let $N \subset M$ be a submanifold of an abstract manifold, and let $Y \in \mathfrak{X}(N)$ be a smooth vector field on N. Prove that there exists an open set $W \subset M$ with $N \subset W$, and a smooth vector field $\tilde{Y}$ on W whose restriction to N is Y. Prove that if N is closed, this can be obtained with $W = M$.

Prove that if a function $f \in C^\infty(N)$ is extended to a smooth function $F \in C^\infty(M)$ (see Exercise 6a-4), then Yf is the restriction of $\tilde{Y}F$.

Prove that if another smooth vector field X on N is extended in the same manner to a vector field $\tilde{X}$ on M, then $[X, Y]$ is the restriction of $[\tilde{X}, \tilde{Y}]$.

6 Prove Lemma 6.5.2.