GEOM2, 2010

P10. Class program for Friday 7/1

Presentation of E4. Besides select from the following exercises

1 Let $f\colon X \to Y$ be a continuous map between topological spaces.

Prove that if X is connected, then the graph $\{(x, f(x) \mid x \in X\}$ is connected in $X \times Y$.

Prove the same statement for pathwise connectedness.

2 Prove that the graph in Example 5.7.5 is connected [Hint: Apply Lemma 5.9 with E equal to the part of the graph with $x > 0$].

Verify that the function f is *not* continuous.

3 Prove that if X and Y are pathwise connected topological spaces, then so is the product $X \times Y$.

Prove the corresponding statement for connected topological spaces [Hint: Apply Lemma 5.7].

4 Let $N \subset M$ be a submanifold of an abstract manifold, and let $f \in C^\infty(N)$. Assume that there exists a locally finite atlas for M.

a) Prove that there exists an open set $W \subset M$ with $N \subset W$, and a function $F \in C^\infty(W)$ such that $F(x) = f(x)$ for all $x \in N$ [Hint: use P7-4 and partition of unity].

b) Assume now in addition that N is closed in M. Prove that the above can be accomplished with $W = M$.

c) Give an example which shows the extra condition in part (b) is necessary [Hint: $1/x$ near $x = 0$]

5 Let M and N be oriented and connected abstract manifolds, and let $f\colon M \to N$ be a diffeomorphism. Prove that either f is orientation preserving at every $p \in M$, or it is orientation reversing at avery $p \in M$ [Hint: Prove that the set where it is orientation preserving is open. For this, apply continuity of the determinant of the matrix df_p, with respect to standard bases for given charts].

6 Show that $\mathbb{R}P^n$ is connected for all n. Show that it is not orientable when n is even [Hint: Consider the composition of π with the antipodal map α of the sphere. Apply P8-3 together with exercise 5 above].