# GEOM2, 2010

## P1. Class program for Friday Nov 19

**1** Consider the parametrized curve

$$\gamma(t) = (-\frac{t}{1+t^3}, -\frac{t^2}{1+t^3})$$

with $t \in ]-\infty, -1[ \cup ]-1, \infty[$. Verify that $\gamma$ is a bijection onto the set

$$\mathcal{C} = \{(x,y) \in \mathbb{R}^2 \mid x^3 + xy + y^3 = 0\}.$$

**2** Let $\gamma$ be as above, with $t \in I$ where $-1 \notin I \subset \mathbb{R}$. In each of the following cases $I = I_i$, prove or disprove that $\gamma$ is an embedded parametrized curve.
1) $I_1 = ]-\infty, -1[$,
2) $I_2 = ]-1, 0[$,
3) $I_3 = ]0, \infty[$,
4) $I_4 = ]-1, \infty[$.

**3** Prove that a singleton set $\{p\} \subset \mathbb{R}^2$ is not a curve.

**4** Let $\mathcal{C} \subset \mathbb{R}^2$ be a curve. Prove that $\mathcal{C}$ has no interior points.

**5** Verify that the set $\mathcal{C}$ in number 1 above is not a curve in $\mathbb{R}^2$.

**6** Let $c \in \mathbb{R} \setminus \{0\}$ and put

$$\mathcal{C} = \{(x,y) \in \mathbb{R}^2 \mid x^3 + xy + y^3 = c\}.$$

Verify that $\mathcal{C}$ is a curve in $\mathbb{R}^2$ if $c \neq \frac{1}{27}$.

**7** Let $c = \frac{1}{27}$ and let $\mathcal{C}$ be as above. Prove or disprove that it is a curve in $\mathbb{R}^2$.