Extra Exercise 1

In this exercise we consider the differential equation

$$\frac{d^3x}{dt^3} = 7\frac{d^2x}{dt^2} - 7\frac{dx}{dt} - 15x \tag{1}$$

on $\mathbf{R} \times \mathbf{R}$ with initial conditions

$$x(0) = 0, \frac{dx}{dt}(0) = -2, \frac{d^2x}{dt^2}(0) = 8. \tag{2}$$

(a) Find three different solutions of (1) by making the Ansatz that $x = e^{\lambda t}$ for some $\lambda \in \mathbf{C}$.

(b) Use these solutions to find a solution to the initial value problem (1) and (2).

Next we will solve the initial value problem in a different way.

(c) Rewrite (1) as a equation of the form

$$\frac{dy}{dt} = Ay,$$

on $\mathbf{R} \times \mathbf{R}^3$, where A is a 3×3-matrix.

(d) What is the initial value condition for y?

(e) Calculate the eigenvalues and eigenvectors of A.

(f) Calculate e^{tA} using Exercise 6.7.

(g) Find the solution to the initial value condition.