Consider the differential equation
\[
\frac{dy}{dt} - 2y \frac{d^2y}{dt^2} = -1.
\]
on \mathbb{R} \times \mathbb{R}.

(a) Prove that there exists a solution \((I, y)\), with \(I\) containing 0 and \(y\) satisfying the initial conditions \(y(0) = 1\) and \(\frac{dy}{dt}(0) = 0\). Prove that this solution is locally unique at \(t = 0\), in the sense that if \((I', y')\) is another solution such that 0 \(\in I'\) and \(y'\) satisfies the initial value conditions, then \(y'\) equals \(y\) on \(I \cap I'\). (Hint: rewrite the differential equation to a first order differential equation and use that \(y(0) > 0\).)

(b) Show that there exists an \(\epsilon > 0\) such that \(y\) is increasing on \((0, \epsilon)\).