Assignment 4

Due Wednesday, March 6.

Consider the differential equation

$$\frac{dy}{dt} - 2y\frac{d^2y}{dt^2} = -1.$$

on $\mathbf{R} \times \mathbf{R}$.

(a) Prove that there exists a solution (I, y), with I containing 0 and y satisfying the initial conditions $y(0) = 1$ and $\frac{dy}{dt}(0) = 0$. Prove that this solution is locally unique at $t = 0$, in the sense that if (I', y') is another solution such that $0 \in I'$ and y' satisfies the initial value conditions, then y' equals y on $I \cap I'$. (Hint: rewrite the differential equation to a first order differential equation and use that $y(0) > 0$.)

(b) Show that there exists an $\epsilon > 0$ such that y is increasing on $(0, \epsilon)$.