Assignment 3

Due Wednessday, February 27.

For $\alpha > 0$, define $X_\alpha : R \times \mathbf{R}_{>0}$ by

$$X_\alpha(t, y) = \frac{\cosh(t)|y^2 - 1|^\alpha}{y} \qquad \big((t, y) \in \mathbf{R} \times \mathbf{R}_{>0}\big).$$

(a) For which $\alpha > 0$ is X_α locally Lipschitz on $\mathbf{R} \times \mathbf{R}_{>0}$?

(b) For which $\alpha > 0$ is X_α Lipschitz on $\mathbf{R} \times \mathbf{R}_{>0}$?

(c) Prove that X_α is locally Lipschitz on $\mathbf{R} \times \big((0, 1) \cup (1, \infty)\big)$.

(d) Let $(t_0, y_0) \in \mathbf{R} \times \big((0, 1) \cup (1, \infty)\big)$. Prove that there exists a solution (I, y) of

$$\frac{dy}{dt} = X_\alpha(t, y) \tag{1}$$

such that $t_0 \in I$ and $y(t_0) = y_0$.

(e) Let (I, y) be the solution in (d). Prove that there exists a neighborhood U of t_0 such that $(I \cap U, y\big|_{I \cap U})$ is the unique solution of (1) on $(I \cap U) \times \mathbf{R}_{>0}$ with $y(t_0) = y_0$.

(f) Discuss how all of this relates to Exercise 1 in the first assignment (which was due February 13).