Assignment 2

Due Wednesday, February 20.

In this assignment we consider the differential equation

$$(1+s^2)\frac{d^2u}{ds^2}+s\frac{du}{ds}-u=\frac{1}{\sqrt{1+s^2}} \tag{1}$$

on $\mathbf{R}\times\mathbf{R}$.

(a) Apply the temporal substitution of variables $s=\sinh t$ to rewrite (1) as the linear inhomogeneous differential equation

$$\frac{d^2x}{dt^2}=x+\frac{1}{\cosh t} \tag{2}$$

on $\mathbf{R}\times\mathbf{R}$, where $x=u\circ\sinh$. Hint: use the identity

$$\cosh^2 t-\sinh^2 t=1. \tag{3}$$

to determine $\cosh t$ in terms of s.

(b) Rewrite (2) as a first order differential equation of the form

$$\frac{dy}{dt}=Ay+b(t) \tag{4}$$

on $\mathbf{R}\times\mathbf{R}^2$, where A is a 2×2-matrix and $b:\mathbf{R}\to\mathbf{R}^2$.

(c) Show that

$$e^{tA}=\begin{pmatrix}\cosh t & \sinh t\\ \sinh t & \cosh t\end{pmatrix}.$$

Hint: Use the power series for $\sinh$ and $\cosh$:

$$\sinh t=\sum_{k\text{ odd }\geq 0}\frac{1}{k!}t^k,\quad \cosh t=\sum_{k\text{ even }\geq 0}\frac{1}{k!}t^k,$$

or use the result from Exercise 6.9.

(d) Find a particular maximal solution of (4). Hint: Use the formula for y_0 in Theorem 6.28, but take the factor e^{tA} outside of the integral. Use that $\int\tanh s\,ds=\ln(\cosh s)$.

(e) Determine all maximal solutions of (4).

(f) Determine all maximal solutions of (1).