Assignment 1

Due Wednesday, February 13.

1. Let $D = \mathbf{R} \times (0,\infty)$ and let $X : D \to \mathbf{R}$ be given by

$$X(t,y) = \frac{\cosh(t)\sqrt{|y^2-1|}}{y}.$$

Let $D_1 = (0,\infty) \times (0,\infty)$ and let X_1 be the restriction of X to D_1.

(a) Give the maximal solution to the initial value problem

$$\frac{dy}{dt} = X_1(y,t), \quad y(1) = \cosh(1).$$

We denote the solution found in (a) by y_1.

(b) Show that

$$(\mathbf{R}, t \mapsto 1)$$
$$\left(\mathbf{R}, t \mapsto \begin{cases} 1 & t \leq 0 \\ y_1(t) & t > 0 \end{cases}\right)$$
$$\left((-\text{arsinh}(1), \infty), t \mapsto \begin{cases} \sqrt{1-\sinh^2(t)} & t \leq 0 \\ y_1(t) & t > 0 \end{cases}\right)$$

are maximal solutions of $\frac{dy}{dt} = X(y,t)$. (Recall that arsinh is the inverse function of $\sinh$.) Conclude that the initial value problem

$$\frac{dy}{dt} = X(t,y), \quad y(0) = 1$$

does not have a unique solution.

2. Let $D = (0,\infty) \times \mathbf{R}$ and let $X : D \to \mathbf{R}$ be given by

$$X(t,y) = \frac{2y}{\sinh(2t)} + \sinh(t)$$

(a) Show that

$$\frac{\tanh'(t)}{\tanh(t)} = \frac{2}{\sinh(2t)}.$$

(b) Give all maximal solutions of

$$\frac{dy}{dt} = X(y,t).$$