

## Matematik 3GE

Written exam, 4 hours. All course material is allowed during the exam (alle sædvanlige hjælpemidler er tilladt).

There are 3 problems divided into 11 questions. All questions are given approximately the same weight.

### Problem 1

- (a) **Find** the regular points of the curve

$$\gamma(t) = \left(\frac{1}{2}t^2, t^3, \frac{1}{2}t^4\right)$$

- (b) **Find** the curvature and torsion of  $\gamma$  in its regular points.

### Problem 2

Let  $S$  denote the surface of revolution given by rotating the parabola  $y = x^2 - 4$  around the X-axis.

- (a) **Find** the part  $S_{reg}$  of  $S$  which consists of regular points.  
(b) **Find** the principal curvatures and the principal directions at the point  $(0, 4, 0)$ .  
(c) **Give** a set of local parametrisations of  $S_{reg}$  which cover the whole surface.  
(d) **Find the first and the second fundamental form** of  $S_{reg}$  in terms of the chosen parametrisation.  
(e) **Prove** that  $S_{reg}$  is diffeomorphic but not isometric to an open subset of the plane  $\{(x, y, 0) \mid x, y \in \mathbb{R}\}$ .

### Problem 3

let  $\mathcal{O}$  be an oriented simple region on a sphere of radius 1. We denote the boundary of  $\mathcal{O}$  by  $\mathcal{G}$  and give it the induced orientation. We assume that  $\mathcal{G}$  is a simple (no selfintersections) piecewise smooth closed curve formed by joining  $n$  distinct points by geodesic arcs.

- (a) **Find** the supremum of possible sums of the internal angles of  $\mathcal{G}$   
(b) Suppose that  $\mathcal{G}'$  is another simple piecewise smooth closed curve obtained by joining by geodesic intervals  $n$  distinct points of the sphere. We assume that  $\mathcal{G}'$  is entirely contained in  $\mathcal{O}$ .  
**Prove** that the sum of internal angles of  $\mathcal{G}'$  is smaller than the one for  $\mathcal{G}$ .  
(c) **Give** a best lower bound on the sum of internal angles of  $\mathcal{G}$ .  
(d) **Prove** that the lowest bound is never attained.