In this note we compare the definitions of a (smooth) surface, as given in Pressley, and in Chapter 7 of the Geom1 notes.

Pressley gives the definition in two steps. Firstly (p. 60) he defines the concept of a *surface*, and secondly (p. 67) that of a *smooth* surface. However, in the rest of the book *all surfaces considered are assumed smooth, even if this is not explicitly mentioned* (see p. 72).

For the purpose of this note, let us call Pressley's concept of a smooth surface a *surface(P)*. The word *surface* is reserved for the concept defined in the Geom1 notes.

Theorem. *Let $\mathcal{S} \subset \mathbb{R}^3$. Then $\mathcal{S}$ is a surface(P) if and only if it is a surface or empty.*

Proof. Assume $\mathcal{S}$ is a non-empty surface(P). By Pressley's definitions there exists for each point $p \in \mathcal{S}$ an open set $U \subset \mathbb{R}^2$, an open set $W \subset \mathbb{R}^3$ with $p \in W$, and a regular smooth map $\sigma: U \to \mathbb{R}^3$, such that σ is a homeomorphism of U onto $\mathcal{S} \cap W$.

Let $q_0 \in U$ be such that $\sigma(q_0) = p$. In order to verify that $\mathcal{S}$ is a surface we must verify condition (4) in the notes. It follows from the fact that σ is a homeomorphism that σ^{-1} is continuous $\mathcal{S} \cap W \to U \subset \mathbb{R}^2$. Thus for each open set $V \subset U$, the set $\sigma(V)$ (which is the preimage of V by σ^{-1}) is open in $\mathcal{S} \cap W$. Hence $\sigma(V)$ is the intersection of $\mathcal{S}$ with some open set $W' \subset \mathbb{R}^3$, as required in (4).

For the converse we assume that $\mathcal{S}$ is a surface, and equip it with an arbitrary atlas (see Section 7.3). Let $p \in \mathcal{S}$ be given and choose a chart $\sigma: U \to \mathcal{S}$ from the atlas, with p in its image. This chart is regular, injective and it satisfies (9) in Corollary 7.4.

We must verify Pressley's condition in Definition 4.1. Let $W \subset \mathbb{R}^3$ be chosen according to (9) with $V = U$, then W is open and has $\mathcal{S} \cap W = \sigma(U)$. The claim is that $\sigma: U \to \sigma(U)$ is a homeomorphism. We already know that σ is bijective and continuous, and it follows from (9) that the inverse is continuous as well. Hence $\mathcal{S}$ is a surface(P). $\square$

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