

Masterclass on Ergodic theory and von Neumann algebras & The Kadison–Singer Problem

Titles and abstracts

Pete Casazza: *The Kadison–Singer Problem in Mathematics and Engineering*

We will see that the famous, intractable 1959 Kadison–Singer Problem in C^* -Algebras is equivalent to fundamental unsolved problems in a dozen areas of research in pure mathematics, applied mathematics and engineering. This has given all these areas common ground on which to interact as well as explaining why each area has volumes of literature on their respective problem. The recent surprising positive solution to this problem by Marcus, Spielman and Srivastava has given each of these areas a powerful new theorem to use in their work.

Lecture 1: *The Kadison–Singer Problem and an Introduction to Frame Theory*

Abstract: We will give a historical introduction to the Kadison–Singer Problem. We will then give an introduction to Frame Theory, the area of research which ties together all the equivalents of KS.

Lecture 2: *The Paving Conjecture, R_ε -Conjecture and the Bourgain–Tzafriri Conjecture*

Abstract: We will introduce the Paving Conjecture, the R_ε -Conjecture and the Bourgain–Tzafriri Conjecture and see that PC implies R_ε implies BT.

Lecture 3: *The Kadison–Singer Problem in Engineering: The Feichtinger Conjecture and the Casazza/Tremain Conjecture*

Abstract: We will give an introduction to Signal Processing and see how the Feichtinger Conjecture arose naturally there. We give an introduction to Internet Coding and see how the Casazza/Tremain Conjecture arose naturally there.

Lecture 4: *The Sundberg Problem, the Kadison–Singer Problem in Harmonic Analysis and in Number Theory*

Abstract: We will see that the Bourgain–Tzafriri Conjecture implies the Feichtinger Conjecture implies the Sundberg Problem. Then we will look at the Kadison–Singer Problem in Harmonic Analysis and in Number Theory.

Lecture 5: *The Weaver Conjectures*

Abstract: We will introduce the Weaver Conjecture KS_r , and the Weaver Conjecture and show that the Bourgain–Tzafriri Conjecture implies KS_r implies the Weaver Conjecture implies the Paving Conjecture.

Lecture 6: *The Proof of the Kadison–Singer Problem*

Abstract: We will look at the Marcus/Spielman/Srivastava proof of the Weaver Conjecture KS_r which we now know implies a positive solution to the Kadison–Singer Problem.

Steven Deprez: *Introduction to the link between von Neumann algebras and ergodic theory*

In this introductory talk, I will sketch a few links between ergodic theory. I will introduce ergodic equivalence relations and the von Neumann algebras related to them. I will concentrate mainly on classical results, like Singer’s theorem and Popa’s intertwining-by-bimodules technique.

Ilijas Farah: *The other Kadison–Singer problem*

It asks whether every pure state on $B(H)$ can be diagonalized by a masa and its apparently stronger (yet equivalent modulo the recent solution to the famous Kadison–Singer problem) version was conjectured by Anderson. One half of this problem was solved by Akemann and Weaver, who showed that the negative answer follows from the Continuum Hypothesis. At present it is not known whether positive answer is consistent with ZFC. In a joint work with Weaver we have shown that in ‘most’ known models of ZFC the answer is negative.

Damien Gaboriau: *Around the orbit equivalence theory of the free groups*

The goal of this series of lectures is to present an overview of the theory of orbit equivalence, with a particular focus on the probability measure preserving actions of the free groups.

I will start by giving the basis of the theory of orbit equivalence and explain the theory of cost. In particular, prove such statements as the induction formula and the computation of the cost of free actions of some countable groups, including free groups. This will be related to the fundamental group of equivalence relations. I intend to present Abert–Nikolov theorem relating the cost of profinite actions to the rank gradient of the associated chain of subgroups. Next, I will consider a recent result of F. Le Maître establishing a perfect connection between the cost of a probability measure preserving action with the number of topological generators of the associated full group. I shall also discuss the number of non orbit equivalent actions of countable groups.

A good part of this material should combine with the lectures delivered by C. Houdayer.

Cyril Houdayer: *Von Neumann algebras with unique Cartan decomposition*

This series of lectures will aim at giving an overview of the recent developments on the uniqueness of Cartan decomposition in von Neumann algebras.

First, I will give a proof of the following result due to S. Popa and S. Vaes (2011): any free ergodic probability measure preserving action of a free group gives rise to a group measure space II_1 factor with a unique Cartan subalgebra, up to unitary conjugacy. Combined with D. Gaboriau's results on cost, this shows that II_1 factors arising from pmp actions of free groups with different ranks are not isomorphic.

Next, I will show how to generalize Popa–Vaes's results to nonsingular actions of the free groups (joint work with S. Vaes). We will then obtain a large family of type III factors with a unique Cartan decomposition, up to unitary conjugacy.