The Kadison-Singer Problem in Mathematics and Engineering

Lecture 3: The Kadison-Singer Problem in Engineering
The Casazza/Tremain Conjecture and the Feichtinger Conjecture

Master Course on the Kadison-Singer Problem
University of Copenhagen

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Signal Processing

The best way to understand signal processing is to attend a piano concert.
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Pictures by: Music With Ease http://www.musicwitthease.com/
To the Pianist

We are hearing a continuous acoustical signal (If we ignore the banging of the keys).
To the Pianist

We are hearing a continuous acoustical signal (If we ignore the banging of the keys).

To the pianist, the concert is a collection of black dots on a piece of paper.

Sheet Music:
In the Audience

If we were fast enough, we could write the sheet music as the concert is being played.
In the Audience

If we were fast enough, we could write the sheet music as the concert is being played.

Then, when we get home, we could use our sheet music to replay (i.e. **reconstruct**) the concert.
Correcting Mistakes?

But what if the pianist made a mistake?
Correcting Mistakes?

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What if she played some notes that did not belong in the concert?
Correcting Mistakes?

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Well, we could just erase the incorrect notes and play back a perfect concert.
Correcting Mistakes?

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What if she played some notes that did not belong in the concert?

Well, we could just erase the incorrect notes and play back a perfect concert.

If she left out some notes, we could just add them.
Dennis Gabor 1946

To Gabor, sheet music was made up of just one note.
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We take that note and change its modulation, then shift it in time and change its modulation and continue.
Our Basic Note
We Change the Modulation of Our Note
Change the Modulation again
Shift our note in time and change modulation
Gabor’s Idea

We will write sheet music for a signal.
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We will write sheet music for a signal.

Fix $g \in (L^2(\mathbb{R}) = \{ f : \mathbb{R} \to \mathbb{C} | \int_{\mathbb{R}} |f(t)|^2 \, dt < \infty \}) \cap L^\infty(\mathbb{R})$—our NOTE
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Fix $0 < a, b \leq 1$. 
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Let $f \in L^2(\mathbb{R})$—our SIGNAL

Fix $0 < a, b \leq 1$.

Take modulations of our “note” and compute the intensity of our signal for each of these:

$$(\langle f, e^{2\pi i a t} g(t) \rangle)_{n \in \mathbb{Z}}$$
Now, take a translation in time of our note and for all modulations of our “translated note”, compute the intensity of our signal for each of these:
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\langle f, e^{2\pi i a t} g(t - b) \rangle_{n \in \mathbb{Z}}
\]
Now, take a translation in time of our note and for all modulations of our “translated note”, compute the intensity of our signal for each of these:

\[
\left( \langle f, e^{2\pi i n t} g(t - b) \rangle \right)_{n \in \mathbb{Z}}
\]

Continuing, we digitalize our signal:

\[
\left( \langle f, e^{2\pi i n t} g(t - mb) \rangle \right)_{m,n \in \mathbb{Z}}
\]
For This to Work We Need

1. Our “digits” are unique to the signal.
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2. We have “fast” reconstruction of the signal from its digits.
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2. We have “fast” reconstruction of the signal from its digits.

This requires that

\[(e^{2\pi i a t} g(t - m b))_{m, n \in \mathbb{Z}}\]

is a frame for \(L^2(\mathbb{R})\) called a **Gabor Frame** and denoted \((g, a, b)\).
What Did Gabor Do?

Gabor used Gaussians $e^{-\pi t^2}$ and $a = b = 1$. Unfortunately for Gabor, this is a case which doesn't work.
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Balian-Low

Balian-Low Theorem

If \((g, 1, 1)\) is a Gabor frame for \(L^2(\mathbb{R})\) then either \(tg(t) \notin L^2(\mathbb{R})\) or \(g' \notin L^2(\mathbb{R})\).
Time Frequency Analysis

Time frequency analysis is the mathematics of signal processing.
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**Major Problem**

Classify all \((g, a, b)\) which give Gabor frames.
Classifying Gabor Frames is a Very Difficult Problem

Theorem (C/Kalton)

Classifying the Gabor Frames of the form \((\chi_E, 1, 1)\)
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**Theorem (C/Kalton)**

Classifying the Gabor Frames of the form $(\chi_E, 1, 1)$

$\iff$

**Littlewood’s Problem**

(1977) Classify the integer sets $\{n_1 < n_2 < \ldots < n_k\}$ so that

$$f(z) = \sum_{j=1}^{k} z^{n_j}$$

does not have any zeroes on the unit circle.
Gabor Frames

Theorem (Rieffel)

If \((g, a, b)\) is a Gabor frame then \(ab \leq 1\).
Gabor Frames

**Theorem (Rieffel)**

If \((g, a, b)\) is a Gabor frame then \(ab \leq 1\).

**Theorem**

If \(ab = 1\) and \((g, a, b)\) is a Gabor frame then it is a Riesz basis.
How do we clean up a signal?

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Every Gabor frame I know can be partitioned into a finite number of Riesz basic sequences. Do you think this is always true?
Feichtinger Conjecture (FC)

Every unit norm frame is a finite union of Riesz Basic Sequences.
Equivalent Formulations

Every unit norm Bessel sequence is a finite union of Riesz basic sequences.
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**Proof:** Given a unit norm Bessel sequence \((\phi_i)\) let \((e_i)\) be an orthonormal basis for \(\mathbb{H}\).
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**Proof**: Given a unit norm Bessel sequence \((\phi_i)\) let \((e_i)\) be an orthonormal basis for \(H\). Then \((\phi_i) \cup (e_i)\) is a unit norm frame.
Equivalent Formulations

Every unit norm Bessel sequence is a finite union of Riesz basic sequences.

Proof: Given a unit norm Bessel sequence \((\phi_i)\) let \((e_i)\) be an orthonormal basis for \(H\). Then \((\phi_i) \cup (e_i)\) is a unit norm frame. Partition this into a finite union of Riesz basic sequences.
Theorem (C/Christensen)

If $ab$ is rational and $(g, a, b)$ is a Gabor frame, then this is a finite union of Riesz basic sequences.
“And again, the internet is not something you just dump something on. It’s not a truck. It’s a series of tubes. And if you don’t understand those tubes can be filled and if they are filled, when you put your message in, it gets in line and its going to be delayed by anyone that puts into that tube enormous amounts of material, enormous amounts of material.”

Ted Stevens, Senator, US Congress
Internet Coding
Internet Coding

error control addressing

information

bits

timing

Goyal/Kovačević/Vetterly
Speeding up the Internet

**Problem:** Can we speed up the internet by encoding with a frame instead of an orthonormal basis?

(Pete Casazza)
Frame Research Center
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Speeding up the Internet

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**Answer:** Maybe!
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Answer: Maybe!

What we need are frames which provide efficient reconstruction after erasures.
An equal norm frame minimizes mean squared error after erasures if and only if it is tight.
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A frame \((\phi_m)_{m \in I}\) is **robust to \(k\)-erasures** if for every \(J \subset I, |J| = k\), the family \((\phi_m)_{m \in I \setminus J}\) is still a frame.
Major Problems

Problem

Find the equal-norm tight frames which are robust to $k$-erasures.

See C/Kovačević/Bodmanm/Paulsen/Heath/Kutyniok/...
Major Problems

**Problem**

Find the equal-norm tight frames which are robust to $k$-erasures.

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**Bigger Problem**

We need low computational complexity.
Major Problems

Problem
Find the equal-norm tight frames which are robust to $k$-erasures.

See C/Kovačević/Bodmanm/Paulsen/Heath/Kutyniok/...

Bigger Problem
We need low computational complexity.

Biggest Problem
We also need good estimates on the behavior of reconstruction operators after erasures as well as accounting for quantization errors.
A Strengthening of KS

The following is a strengthening of Weaver’s Conjecture.
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C/Tremain Conjecture

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There is a universal constant $0 < A$ and an integer $K > 2$ so that every unit norm $K$-tight frame $\{\phi_i\}_{i=1}^{KN}$ for $\mathbb{H}_N$ can be partitioned into two subsets.
A Strengthening of KS

The following is a strengthening of Weaver’s Conjecture

C/Tremain Conjecture

There is a universal constant $0 < A$ and an integer $K > 2$ so that every unit norm $K$-tight frame $\{\phi_i\}_{i=1}^{KN}$ for $\mathbb{H}_N$ can be partitioned into two subsets each of which have lower frame bound $A$. 
C/Tremain Conjecture

That is:

there is a universal constant $A > 0$
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there is a universal constant $A > 0$ so that if

$$\| \phi \|_2^2 = \sum_{i=1}^{KN} |\langle \phi, \phi_i \rangle|^2, \text{ for all } \phi \in \mathbb{H},$$
C/Tremain Conjecture

That is:

there is a universal constant $A > 0$ so that if

$$\|\phi\|^2 = \sum_{i=1}^{KN} |\langle \phi, \phi_i \rangle|^2, \text{ for all } \phi \in H,$$

then we can find $J \in \{1, 2, \ldots, KN\}$ so that for all $\phi \in H$, 

$$A \|\phi\|^2 \leq \sum_{i \in J} |\langle \phi, \phi_i \rangle|^2 \text{ and } A \|\phi\|^2 \leq \sum_{i \in J^c} |\langle \phi, \phi_i \rangle|^2.$$
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**Important:** $A, K$ must be independent of $N$. 
[C/Tremain Conjecture]

There is some $K$ so that if $\left(\phi_i\right)_{i=1}^{KN}$ are unit norm vectors in $\mathbb{H}_N$.
There is some $K$ so that if $(\phi_i)_{i=1}^{KN}$ are unit norm vectors in $\mathbb{H}_N$ satisfying

$$\sum_{i=1}^{KN} \phi_i \phi_i^* = K \cdot I,$$

then there is a subset $J \subset \{1, 2, \ldots, KN\}$ so that

$$a \cdot I \leq \sum_{i \in J} \phi_i \phi_i^*$$

and

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Restated Again

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$$
Equivalently

Note that

\[ A\|\phi\|^2 \leq \sum_{i \in J} |\langle \phi, \phi_i \rangle|^2 \]
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\[ A \| \phi \|^2 \leq \sum_{i \in J} |\langle \phi, \phi_i \rangle|^2 \]

\[ = \sum_{i=1}^{KN} |\langle \phi, \phi_i \rangle|^2 - \sum_{i \in J^c} |\langle \phi, \phi_i \rangle|^2 \]
Equivocally

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if and only if

\[ \sum_{i \in J^c} |\langle \phi, \phi_i \rangle|^2 \leq (K - A) \| \phi \|^2 . \]
An equivalent formulation of the conjecture is:

There is a universal constant $A > 0$ so that whenever

\[
\| \phi \|_2^2 = K N \sum_{i=1}^N |\langle \phi, \phi_i \rangle|^2,
\]

for all $\phi \in H$, then we can find $J \in \{1, 2, \ldots, KN \}$ so that for all $\phi \in H$,

\[
\sum_{i \in J} |\langle \phi, \phi_i \rangle|^2 \leq (K - A) \| \phi \|_2^2
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\]

Remark: It was shown that the CT conjecture implies the Kadison-Singer Problem. But we did not know if they were equivalent. We thought that CT might actually be formally stronger than KS.
C/Tremain Conjecture

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But we did not know if they were equivalent.
We thought that CT might actually be formally stronger than KS.
A form of CT Equivalent to KS

There exists a $\delta > 0$ and a natural number $r$ so that:

$$\sum_{i} \phi_i \phi_i^* \leq (1 - \delta) \cdot I.$$
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for all large $K$ and all equal norm Parseval frames $(\phi_i)_{i=1}^{KN}$ in $\mathbb{H}_N$
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$$\sum_{i=1}^{KN} \phi_i \phi_i^* = I,$$
There exists a $\delta > 0$ and a natural number $r$ so that:
for all large $K$ and all equal norm Parseval frames $(\phi_i)_{i=1}^{KN}$ in $\mathbb{H}_N$ I.e.
\[
\sum_{i=1}^{KN} \phi_i \phi_i^* = I,
\]
there is a partition $(A_j)_{j=1}^r$ of $\{1, 2, \ldots, Kn\}$ so that
\[
\sum_{i \in A_j} \phi_i \phi_i^* \leq (1 - \delta) \cdot I.
\]
Recent Announcement

A consortium led by Muriel Medard, a Professor at MIT’s Research Laboratory of Electronics and a leader in the effort, includes researchers at MIT, the University of Porto in Portugal, Harvard University, Caltech, and the Technical University of Munich is licensing a new technology designed to deal with lost packets (erasures) during wireless transmission and expects this to be a quantum leap forward in the area. This work rests on doing reconstruction in packet based wireless networks after erasures.