

Corrections to “Classification of Nuclear, Simple C^* -algebras”

- **p. 11, l. -12:** It is not true in general that multiplier inner automorphisms are approximately inner. However, it is true whenever A is of stable rank one or whenever A is stable. A counterexample to the general statement is obtained eg. by taking $A = C_0(\mathbb{R}^+, B)$, where B is the Bunce-Deddens algebra, and the automorphism on A is conjugation by a unitary u in B whose class in $K_1(B)$ is a generator.
- **p. 26, l. 6:** Replace “or A admits an approximate unit consisting of projection” by “or $A \otimes \mathcal{K}$ admits an approximate unit consisting of projection”.
- **p. 33, l. 2:** Replace “ $\varphi_{k+1} \circ \psi_k \circ \varphi_k$ ” with “ $\varphi_{k+1} \circ \psi_{k+1} \circ \varphi_k$ ”.
- **p. 87, l. -4:** It is not true that the C^* -algebras $C^*(v_1), C^*(v_2), \dots, C^*(v_r)$ commute with each other. Actually, v_i and v_j anticommute. It is nevertheless true that $C^*(v_1, v_2, \dots, v_r)$ is isomorphic to $M_2 \otimes M_2 \otimes \dots \otimes M_2$ and that the isomorphism σ acts as claimed.
- **p. 93, l. -10:** Replace the reference “[84, Lemma 1.9]” with “[84, Lemma 1.6]”.
- **p. 95, l. 14:** Replace the reference “[84, Lemma 1.8]” with “[84, Lemma 1.5]”.
- **p. 96, l. 5:** Replace “such that $|x_n| < \varepsilon$ ” with “such that $|x_n - x_0| < \varepsilon$ ”.
- **p. 97, Lemma 6.2.5:** Assume in and above Lemma 6.2.5 that B is a *unital* C^* -algebra. In the proof of Lemma 6.2.5 replace A_ω by B_ω (p. 97, l. -5), $\ell^\infty(A)$ by $\ell^\infty(B)$ (p. 97, l. -4), A by B (p. 98, l. 2), and $\ell^\infty(A)$ by $\ell^\infty(B)$ (p. 98, l. 3).
- **p. 100, l. 13:** Replace the reference “[84, Proposition 1.7]” with “[84, Proposition 1.4]”.
- **p. 101, l. 13:** Replace three occurrences of “ $\varphi(1_A)$ ” with “ $\varphi(1_{M_n(\mathbb{C})})$ ”.
- **p. 101, l. 17:** Replace the reference “[84, Lemma 1.10]” with “[84, Lemma 1.7]”.
- **p. 103, l. -11:** The second “=” in the display should be replaced with “ \leq ”.
- **p. 105, Lemma 6.3.9:** In the lemma and its proof we must replace $(\mathcal{O}_2)_\infty$ with $(\mathcal{O}_2)_\omega$ for some free ultrafilter ω . In the proof of the lemma (eg. when one has to establish the inequalities on p. 106, l. 5) it is important that the norm of $\pi_\omega(x)$, where x is an element in $\ell^\infty(\mathcal{O}_2)$, is an actual limit: $\lim_\omega \|x_n\|$, rather than $\limsup_{n \rightarrow \infty} \|x_n\|$. Consequently, in the proof of Theorem 6.3.11 on page 108 we must similarly change all occurrences of $(\mathcal{O}_2)_\infty$ with $(\mathcal{O}_2)_\omega$. One must also modify the proof of Lemma 6.3.10: Using standard facts about quasidiagonal C -algebras, one can show that

the embedding φ can be chosen such the composition of the embedding $\bar{t} \circ \varphi$ with the natural surjection $(\mathcal{O}_2)_\infty \rightarrow (\mathcal{O}_2)_\omega$ yields an embedding $A \rightarrow (\mathcal{O}_2)_\omega$. Now the (revised) Lemma 6.3.9 applies

- **p. 105, l. -1:** Replace “operator spaces” with “operator systems”.
- **p. 107, l. -8:** The crossed product $C_0(\mathbb{R}, A) \rtimes_\tau \mathbb{Z}$ is isomorphic to $\mathcal{K} \otimes C(\mathbb{T}) \otimes A$, not to $\mathcal{K} \otimes A$. It remains true that A embeds into $\mathcal{K} \otimes C(\mathbb{T}) \otimes A$ and hence into $C_0(\mathbb{R}, A) \rtimes_\tau \mathbb{Z}$.

Actually, one can obtain this embedding in a nicer way: One observes that the crossed product $C_0(\mathbb{R}, A) \rtimes_\tau \mathbb{Z}$ is isomorphic to $(C_0(\mathbb{R}) \rtimes_\tau \mathbb{Z}) \otimes A$, and that $C_0(\mathbb{R}) \rtimes_\tau \mathbb{Z}$ contains a non-zero projection p . One can then take the embedding to be: $a \mapsto a \otimes p$.

The (Rieffel) projection p is given by $p = gu^* + f + ug$, where u (in the multiplier algebra of $C_0(\mathbb{R}) \rtimes_\tau \mathbb{Z}$) is the canonical unitary that implements the action τ and $f, g \in C_0(\mathbb{R})$ are given by

$$f(t) = \begin{cases} 1 - |t|, & -1 \leq t \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad g(t) = \begin{cases} \sqrt{t - t^2}, & 0 \leq t \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- **p. 109, l 15:** One must also show that the relative commutant $A_\omega \cap A'$ is different from \mathbb{C} . This follows from the fact the the isometry $s \in A_\omega \cap A'$ can be chosen to be non-unitary. This, again, follows from the fact that the isometry t in Lemma 6.3.2 actually is non-unitary (by construction). It follows that the isometry $s = ut$ from Proposition 6.3.3 likewise is non-unitary. It finally follows from the proof of Corollary 6.3.5 (ii) (second part) that the isometries s_n are non-unitary, finally making the isometry s in the proof of Proposition 7.1.1 non-unitary.
- **p. 112-113, Proposition 7.2.5:** The proof given of this proposition only works when A is also assumed to be nuclear. (The result by Phillips and Lin holds in the stated generality.) More specifically, in the proof of the ”general case” on page 113 one needs nuclearity of A to apply Proposition 7.1.1.

Fortunately, Proposition 7.2.5 is applied to the nuclear C^* -algebra $A = \mathcal{O}_\infty \otimes \mathcal{O}_\infty$ in the proof of Theorem 7.2.6.

- **p. 124, Lemma 8.2.13:** Replace “The map $\kappa_{A,B}: KK(A, B) \rightarrow \widehat{H}(A, B)$..” with “The map $\kappa_{A,B}: \widehat{H}(A, B) \rightarrow KK(A, B \otimes \mathcal{O}_\infty)$..”
- **p. 127, Theorem 8.3.3 (iii):** The two unital, nuclear $*$ -homomorphisms φ and ψ must also be *injective*. In (b), one must replace $\varphi \sim_h \psi$ with *stable homotopy*, i.e., that φ and ψ are homotopic as maps $A \rightarrow B \otimes \mathcal{K}$ (viewing B as a sub-algebra of $B \otimes \mathcal{K}$).

- **p. 132, l. -6:** Replace the last two lines of Corollary 8.4.11 (i) by:

$$(M_{k_1}(\mathcal{O}_{n_1}) \oplus M_{k_2}(\mathcal{O}_{n_2}) \oplus \cdots \oplus M_{k_r}(\mathcal{O}_{n_r})) \otimes C(\mathbb{T}),$$

for some $r \in \mathbb{N}$, some $k_1, k_2, \dots, k_r \in \mathbb{N}$, and some n_1, n_2, \dots, n_r in $\{2, 3, \dots\}$.

Question 8.4.4 on page 129 has been answered in the negative in a revised version of [122].