Beyond classification of Cuntz-Krieger algebras

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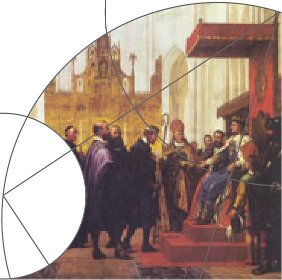
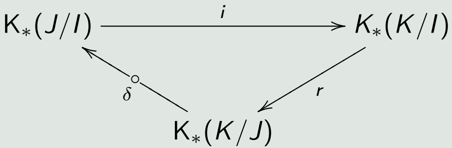


Theorem (Restorff)

Filtrated K*-theory* $\overline{\mathsf{FK}}$ *stably classifies Cuntz-Krieger algebras satisfying condition (II).*

Definition

Filtrated K-theory $\overline{\mathsf{FK}}(A)$ of A consist of all groups $\mathsf{K}_*(J/I)$ with $I \subseteq J$ ideals in A, together with all maps



induced by $J/I \hookrightarrow K/I \twoheadrightarrow K/J$ when $I \trianglelefteq J \trianglelefteq K \trianglelefteq A$.

- Does $\overline{\mathsf{FK}}$ strongly classify the Cuntz-Krieger algebras up to stable isomorphism?
- Does $(\overline{\mathsf{FK}}, [1_A])$ classify the Cuntz-Krieger algebras up to unital isomorphism?
- Does $\overline{\mathsf{FK}}$ stably classify the purely infinite graph C^*-algebras with a finite ideal lattice?
- Let A be a purely infinite, separable, nuclear C^*-algebra in the bootstrap class and with a finite ideal lattice.
  If $\overline{\mathsf{FK}}(A) \cong \overline{\mathsf{FK}}(B)$ for B a Cuntz-Krieger algebra, is A then stably isomorphic to a Cuntz-Krieger algebra?

Theorem (Kirchberg)

Let A and B be O_∞-absorbing, separable, nuclear C^*-algebras with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B) \cong X$. Then any $\mathrm{KK}(X)$-equivalence between A and B lifts to an X-equivariant isomorphism between $A \otimes \mathbb{K}$ and $B \otimes \mathbb{K}$.

Theorem (Meyer-Nest)

For X finite linear space, the UCT holds, i.e. the sequence

$$\mathrm{Ext}(\mathrm{FK}_X(A), \mathrm{FK}_X(B)) \hookrightarrow \mathrm{KK}_*(X; A, B) \twoheadrightarrow \mathrm{Hom}(\mathrm{FK}_X(A), \mathrm{FK}_X(B))$$

is exact for all separable C^*-algebras A and B over X where A lies in the bootstrap class $\mathcal{B}(X)$.

Theorem (Bentmann)

For X finite accordion space, the UCT holds.

Corollary

For purely infinite, separable, nuclear C^*-algebras A and B with all simple subquotients in the bootstrap class and with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B) \cong X$ finite accordion space, any isomorphism between $\mathrm{FK}_X(A) = \overline{\mathrm{FK}}(A)$ and $\mathrm{FK}_X(B) = \overline{\mathrm{FK}}(B)$ lifts to an X-equivariant isomorphism between $A \otimes \mathbb{K}$ and $B \otimes \mathbb{K}$.

- Does $\overline{\mathsf{FK}}$ strongly classify the Cuntz-Krieger algebras up to stable isomorphism?
- Does $\overline{\mathsf{FK}}$ stably classify the purely infinite graph C^*-algebras with a finite ideal lattice?
- Let A be a purely infinite, separable, nuclear C^*-algebra in the bootstrap class and with a finite ideal lattice.
  If $\overline{\mathsf{FK}}(A) \cong \overline{\mathsf{FK}}(B)$ for B a Cuntz-Krieger algebra, is A then stably isomorphic to a Cuntz-Krieger algebra?

Theorem (Meyer-Nest)

There exist a finite space X_0 *and purely infinite, separable, nuclear* C^**-algebras* A *and* B *with* $\mathrm{Prim}(A) \cong \mathrm{Prim}(B) \cong X_0$ *and* $A, B \in \mathcal{B}(X_0)$ *satisfying that*

$$\mathrm{FK}_{X_0}(A) \cong \mathrm{FK}_{X_0}(B), \quad \mathrm{KK}_*(X; A, B)^{-1} = \emptyset.$$

Theorem (Meyer-Nest)

For the space X_0 there exists a functor FK' for which the sequence

$$\mathrm{Ext}(\mathrm{FK}'(A), \mathrm{FK}'(B)) \hookrightarrow \mathrm{KK}_*(X_0; A, B) \twoheadrightarrow \mathrm{Hom}(\mathrm{FK}'(A), \mathrm{FK}'(B))$$

is exact for all separable C^*-algebras A and B over X_0 where A lies in the bootstrap class $\mathcal{B}(X_0)$.

Theorem (A-Restorff-Ruiz)

For C^*-algebras A and B over X_0 with A of real rank zero, any isomorphism between $\mathrm{FK}_{X_0}(A)$ and $\mathrm{FK}_{X_0}(B)$ lifts to an isomorphism between $\mathrm{FK}'(A)$ and $\mathrm{FK}'(B)$.

Corollary

For purely infinite, separable, nuclear C^*-algebras A and B with all simple subquotients in the bootstrap class, with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B) \cong X_0$ *and with A of real rank zero, any isomorphism between* $\mathrm{FK}_{X_0}(A) = \overline{\mathrm{FK}}(A)$ *and* $\mathrm{FK}_{X_0}(B) = \overline{\mathrm{FK}}(B)$ *lifts to an X_0-equivariant isomorphism between* $A \otimes \mathbb{K}$ *and* $B \otimes \mathbb{K}$.