Practical Optimization in Finance, Fall 2001

Suggestion for Project 3: Term Structure Fitting & Pricing of Mortgage Products

Calibrate a BDT-tree to the term structure. (On the file `danish.xls` you can find 10 years of estimated Danish term structure. But feel free to use something else.) Assume for simplicity that “the short rate” means the 1-year ZCB spot rate; work with time-steps of 1 year. Make a qualified guess – perhaps even an estimate – of the $b$-parameter.

One Thing to do (One available product)
Find the price of a 30-year bullet bond where the coupon payment each year is fixed at the short rate. (A so-called floating rate note; FRN) How does this price depend on the volatility of the short rate (the parameter $b$ in the BDT-fit).

Suppose that FRN is “capped” at some level $\bar{r}$. This means that the coupon payment at any coupon date $t$ is $\min(r_{i-1}, \bar{r})$. Find prices of capped FRN for different $\bar{r}$-levels. What about $b$-dependence now?

Do the analysis also for a “floating rate annuity”, which is to be understood as follows: At time $t_i$ the repayments of principal, $A_i$, (“A” is for “Afdrag”) are fixed as if it were fixed-rate annuity with coupon-rate $r_0$. (Recall – from high school if not since – the annuity formula that says that the (constant) installment for an $N$ year annuity with coupon rate $c$ is $(c(1+c)^N)/((1+c)^N-1)$.)

The installment at each time-point $t_i$ is then $P_{i-1} \ast r_{t_i} + A_i$, where $P_{i-1}$ is the remaining principal outstanding at time $t_{i-1}$ (so $P_i = P_{i-1} - A_i$).

Another Thing to do (The callable bond)
Most homes in Denmark are financed by callable fixed-rate annuities. This means that the home-owner can prepay the remaining outstanding principal at any time. (Subtle lingo-point: In Danish such bonds are termed “konverterbare”. That translates to “callable” – “convertible bonds” also exist but are something different.) In David Lando’s note on the homepage (the file `famoes.pdf`
– it’s in Danish, sorry) it is described how to value a callable bond by working backward through an short-rate lattice. Read & implement this. Start with his toy-example (fairly short-termed bullet bond & and a made-up lattice) and work your way up to pricing an annuity in the calibrated model.