Practical Optimization in Finance, Fall 2001

Suggestion for Project 3: Working with Data

The generic way of building stock-price lattices so far has been to put

\[ u_n = 1/d_n = \exp(\sigma/\sqrt{n}) \],

where the volatility, \( \sigma \), is estimated from the standard deviation of stock(-log)-returns.

First, try to estimate parameters from data. (The spreadsheet dailySP500.80to01.xls on the course homepage is a good way to start if you want to do things in Excel.) Try different stock(-indices), different sampling frequencies (monthly, weekly, daily), how many days are there in a year?), different observation periods. If you’re really fancy you can try (exponential) weighting. This works as follows: Fix some initial period over which \( \sigma \) is first estimated (call this \( \sigma_0 \)). Then update your \( \sigma \)-estimate through time by

\[ \sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) \frac{R_t^2}{\Delta t}, \]

where \( R_t^2 \) is squared log-return and \( \lambda \) is some number between 0 and 1. (Try 0.9-0.99.)

When \( n \to \infty \) this becomes the Black-Scholes model, where stock-returns are normally distributed. While this is by no means the worst approximation, it may not be completely satisfactory either. Try to investigate the normality assumption; There’s no need to do any formal tests, but try to compute 3. and 4. order moments for the returns & try to plot a histogram of the observed returns (against a normal one).

One Thing to do

There are many ways to try to remedy non-normality of returns. One is described in Mark Rubinstein’s article “As Simple As One, Two, Three”, that you can find on the course homepage (in Word-format). Read it & implement it. (Remarks: Rubinstein works completely in \( q \)-probability terms (so his \( p \)'s are our \( q \)'s), he has in effect scaled the initial stock price to 1, and uses \( r \) as
“1+ the interest rate”.) More specifically, focus on the section called “The
Solution”. Try to build, say, a 5-period binomial model where the terminal
return distribution mimics the empirical distribution of monthly S&P-500 re-
turns. (Here you have choices to make regarding cumulative probabilities of
& values at end nodes. Use a histogram to make inspired choices.)
Compare option-prices to those in a “standard” lattice (with $\sigma$ estimated &
the same risk-free rate $r$ in Rubinstein’s tree.)

Another Thing to do
You could also try to see how bad you’re off, empirically, if you hedge with the
simple binomial model. “If it ain’t broke, why fix it?” To this end consider
the following plan:

- Download a long series of daily stock-price observations. Calculate log-
returns & subtract the mean log-return from all observations (just for
simplicity). Index these in the rather fancy way $(\tilde{R}_t)_{t=180}^{N+21}$.
- Pick a random $j \in \{0, 1, \ldots, N\}$
- Use the “window” $\tilde{R}, \tilde{R}_{j-1}, \ldots, \tilde{R}_{j-180}$ to estimate volatility, $\sigma$ “for use in
a binomial model”.
- Suppose that, standing at time $j$, you want to hedge a call-option with
21 days to expiry. You fully believe in the binomial model, so you use
the estimated $\sigma$ to build one & set up your hedge position. (Assume that
$S(0) = \text{strike} = 1$. Use your “Project 2” program for this calculation.)
- Now one day passes, hence we are at time $j+1$ & the option in focus
has 20 days to expiry. You observe a stock-price of

$$S_1 = S_0 \exp(\tilde{R}_{j+1}).$$

You still firmly believe in the binomial model & set up a new hedge-
position & use the bank account for adjustments (assume $r = 0$). Keep
track the value of the portfolio.
- Do this all the way out to $i = j+20$ (recursively putting $S_i = S_{i-1} \exp(\tilde{R}_{j+i})$).
Determine the terminal value of the portfolio & compare it to the call-
option pay-off ($(S_{j+21} - 1)^+$). How well did the replication work?
- Now pick another $j$ and do the whole thing again. And again. And ...