Practical Optimization in Finance, Fall 2001

"Aftermath" of Lectures on Wednesday November 14, 2001

Projects: Suggestions for final projects have now been posted on the course home-page.

The Lectures November 14: Having shown last time how to price zero-coupon bonds (and other, more advanced, things) in a stochastic lattice-model for the short term interest rates (the usual "working backwards"-way), I turned to the more advanced inverse question: Given an observed term-structure (for instance estimated by the method you used in the first project) how can I construct a stochastic short rate model (i.e. an r-lattice) that is consistent with this observed term-structure? By consistent I mean that if I work my way backward through the r-lattice & find theoretical prices of zero-coupon bonds, THEN these are equal to the observed one. It is important to understand that this is our objective: to match a dynamic model (which is largely something "we create in our heads") to the hard facts (observed prices). We could just try with very-brute-force trial and error: Make a guess of an r-lattice. See if we made a correct guess. If not, guess again. If there are $T$ observed ZCB prices, then our problem is one of solving $T$ (non-linear) equations with $T^2$ unknowns. So many solutions exist. (Surprisingly often people get non-uniqueness mixed up with non-existence. Don't.) We could therefore put a lot of extra structure on the problem; see how in Section 8.2.1 in the "other course notes" on the homepage. In fact, we ended up with $T$ equations in $T$ unknowns that could be solved recursively as $T$ equations each with 1 unknown. The time needed to solve these was of the order $T^3$. A further refinement (known as the forward fitting algorithm) could bring this down to $T^2$. This looked, and was, advanced, but easy to implement in a spreadsheet. I showed how & you can relive the action on the course home-page.

Finally I discussed one application of an r-lattice, namely pricing of callable bonds ("konverterbare lån"), i.e. loans where you can choose to prepay the remaining outstanding principal at any time. The trick was to work backwards through the lattice, at each point checking which is cheaper: Prepaying the principal or keeping the loan (the "cost" of which is found as the $Q$-expected discount value on the loan at the two future nodes). And doing what costs you less. I gave an example in a spreadsheet (the same one as the forward fitting algorithm) & and you can read about it in David Lando note (in Danish) that you can find on the course home-page.

Kindly,

Rolf