Practical stuff
The lectures on Monday November 29 and the exercises Tuesday November 30 have been switched (but the same rooms are used).

Past lectures
Monday November 22: Chapter 22 “except all the stuff on calibration”. Reducing the calculation of ZCB prices to solving ODEs, as is done in the affine models, is very useful. The Vasicek model can be treated by “brute force” (a cumbersome exercise we used to do in the old days), for the Cox-Ingersoll-Ross model, it’s next to impossible (solving the ODE requires a sharp pencil, but is a doable exercise).
Barring time-dependent coefficients, the statement in Proposition 22.2 is “if and only if”: We have an ATS if and only if $\mu$ and $\sigma^2$ are affine in $r$. (This is a small exercise.) Further, the result can be generalized to higher dimensions. (A somewhat bigger exercise; there are details in Darrell Duffie and Rui Kan, A Yield factor Model of Interest Rates, Mathematical Finance, vol. 6, pp 379-406.)
The explicit solution for the Vasicek model also comes in handy time and again. For instance below.

Wednesday November 24: Done with Chapter 22. Vasicek on real data. Exercise 12.3 walks you through what I did. Calibration. Exercise 12.4 walks you through what I should have done when covering section 22.4.2. Section 22.4.4 does the same for the Vasicek. I’d didn’t do the details.

Coming attractions
Starting Tuesday November 30 (and continuing on Wednesday December 1) we’ll look at Chapter 24 on so-called change of numeraire techniques. What they are really good for is pricing options in models with stochastic interest rates. That will be the last “big topic” in the course. Should take a couple of weeks.
Questions for exercises below.
Kindly,

Rolf

Exercises for week 49 (Monday November 29)

Exercise 12.1: Vasicek calculations
Björk’s exercise 22.1, except the last question.

Exercise 12.2: Risk-premia in the Vasicek model
On the homepage, you’ll find an exercise (from the Björk-based PhD-course, that you have met before) that studies risk-premia in the Vasicek model. Solve the first part (which is more ’looking at equations’ than actual calculations), and use it as warm-up to/explanation for Exercise 4 from the 2003/4 MathFin-exam.

Exercise 12.3: Vasicek and data
At the lectures on Wednesday November 24, I spend a lot of time looking at US-data and the Vasicek model. On the homepage, you can find

- 1952-2004 US interest rate data (ie. I updated relative to what I showed the lectures)
- My R-source-code. It is reasonably well-commented (and R-syntax shouldn’t be too off-putting), and “ready to run”.
- Some pretty pictures, I case you “don’t do R”

Get to grips with what the code does and discuss the results.

Points to consider:

- What is the (log)likelihood function for observations from an “Vasicek (under P)” or Ornstein/Uhlenbeck-process? (Use 12.1.) Are the closed-form maximum likelihood estimates correct? (Some differentiation after convenient reparametrization.)
- How is the risk-premium estimated? (Use 12.2.)
Open questions, by which I mean things “that the code doesn’t do”, and I don’t expect you to do either, but are healthy to think about.

- Why are we so fond of maximum likelihood estimators? And can we say something about the standard errors/confidence intervals of our parameter estimates?

- What would happen if we tried risk-premia of the form

$$\lambda(r(t), t) = \lambda_0 + \lambda_1 r(t)?$$

- Can we fix the slight discrepancy that arises for short maturities, because we sometimes we think of 3 months as ‘$dt$’, and sometimes don’t.

**Exercise 12.4: Ho/Lee calibration**

Björk’s exercise 22.5. At the lectures, I screwed up with the signs; too many minuses to keep track of. With some assistance, I got it sorted out afterwards. I promised to put it on the weekly note. And so I did! (The signs on page 334 in Björk are right.)

By the way: Is the “Chapter 22’-Ho/Lee model really the same as the ”Chapter 23’-Ho/Lee model?