Past lectures

**Tuesday November 9:** Barrier options. Up to page 6 in “Exotic Options: Proofs without Formulas”. This replaces chapter 18 from Björk. (For those of you taking notes in the second lecture: Sorry about the apparent “− to +” correction. It meant what I first wrote, $f(x,t) = \Phi(... - \sigma^2/2...)$; just as on page 6 in the note.)

Incomplete models/models for non-traded objects: Sections 15.1 and 15.2 from Björk

**Wednesday November 10:** The rest of from Björk’s chapter 15.

Remarks on exercise 2 from note 9 (see slides on homepage).

Models with stochastic interest rates: We got to the middle of Björk’s definition 20.2.

Coming attractions

More from chapter 20. Once we prove the technical but extremely useful Proposition 20.5, we’ll probably jump briefly to chapter 23 to get some of the benefits, but besides that we continue along the beaten path: 20, 21, 22.

Guess where the questions for the exercises are.

Kindly,

Rolf

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**Exercises for week 47 (Tuesday November 16)**

**Exercise 10.1 Barrier options; the easy case**

Consider down-and-out call-option with barrier $B$, ie. in the notation from page 4 of “Proofs without Formulas”

$$v(x) = (x - K)^+,$$
where we assume $B < S(0)$ and $B < K$.

What does the $g$-function (as defined in the second displayed equation on page 4 in the note) look like. Recall that the $f$-function is defined via

$$f(S(t), t) = \mathbb{E}_t^Q(g(S(T))).$$

What is $f$ in this case?

What is the $\hat{g}$-function and what does the adjusted pay-off function $h$ look like?

Write down (implement!) an explicit formula for the price of the down-and-out call-option. (It should match Björk’s equation (18.17))

**Exercise 10.2: Barrier options; the not-so-easy case**

We now look at a down-and-out put option, ie. in the notation of the note and the previous exercise

$$v(x) = (K - x)^+,\]$$

where we again assume $B < \min(S(0), K)$. What does the $g$-function, $g(x) = (K - x)^+ 1_{x>B}$ look like?

Explain how this $g$-function can be written as a sum of “more ordinary” options. (You’ll need the strike $K$-puts and strike $B$-puts, as well as strike-$B$ digital options.) Argue that given the work you have previously done in this course, this in principle gives us the relevant $f$-function easily, and thus the price of the barrier option. (Stick to principles; things become long/messy when written out in full.)

What do the $\hat{g}$- and $h$-functions look like?

**Exercise 10.3: Payback**

Suppose we want borrow $1 today and pay back a known amount in 1 year. We know that there is precisely one way to do this. Suppose $P(0; 1) = 0.95$. How much do you pay back in 1 year?

Suppose $P(0; 1) = 1/1.05$. How much do you pay back in 1 year?

Suppose the continuously compounded zero coupon yield is 0.05 (ie. 5%). How much do you pay back in 1 year?

Suppose the 1-year (spot) LIBOR, $L(0; 0, 1)$ from definition 15.2(1), is 5%.

How much do you pay back in 1 year?

Suppose all the (relevant) 3-month forward LIBOR rates are 5%,

$$L(0; 0, 0.25) = L(0; 0.25, 0.5) = L(0; 0.5, 0.75) = L(0; 0.75, 1) = 0.05.$$  

How much do you pay back in 1 year? (And how can you tell?)
Can you determine how much is paid back if you are told that $i) r(0) = 0.05\%$, $ii) f(0, 1) = 0.05\%$  $iii) f(0, T) = 0.05$ for all $T \in [0; 1]$?

Exercise 10.4: Defining instantaneous forward rates
At the lectures we saw that the “$S \to T$”-limit of the simple forward rates $L(t; S, T)$ in Definition 20.2(1) does indeed give instantaneous forward rates as

$$f(t, T) = -\frac{\partial \ln P(t, T)}{\partial T}.$$  

Show that we get the same by taking the limit of the continuously compounded forward rates $R(t; S, T)$ defined by

$$\frac{P(t, S)}{P(t, T)} = e^{(T-S)R(t; S, T)}.$$