Futures Options

Consider the “base-case” Black-Scholes model,

\[
\begin{align*}
    dS(t) &= rS(t)dt + \sigma S(t)dW^Q(t), \\
    d\beta(t) &= r\beta(t)dt.
\end{align*}
\]

Consider a futures contract with delivery date \( T \). Let \( F(t; T, S) \) denote its futures price. Find a SDE for the futures price, i.e. find \( dF(t; T, S) \).

Consider a futures (call) option with expiry date \( T^* < T \) and strike \( K \). This is a call option on the futures price, i.e. it pays off \( (F(T^*; T, S) - K)^+ \) at time \( T^* \).

Find a formula for the time-\( t \) arbitrage-free price of the futures option, say \( \pi^{\text{FUTOpt}}(t) \).

How do things change if the stock pays a dividend yield of \( \delta \)?

How can you hedge the futures option \( i) \) with stock? \( ii) \) with the futures contract?