Extending Theory

Consider the base-case lognormal Black-Scholes model, i.e.
\[
\begin{align*}
&dS(t) = \alpha S(t) dt + \sigma S(t) dW^P(t), \\
&d\beta(t) = r\beta(t) dt.
\end{align*}
\]

Imagine that the bank where you work has developed a new product: The **extendible call-option**. This is an option where the buyer can do one of two things at maturity date \(T_1\):

- Get the pay-off of an ordinary strike-\(K_1\) call-option, i.e. \((S(T_1) - K_1)^+\).
- Extend the option. This means that by paying a predetermined amount to the seller, \(A\), at time \(T_1\) he receives a new (standard) call option with maturity date \(T_2\) and strike \(K_2\). (So both \(A\), \(T_2\), and \(K_2\) are known/specified at initiation of the contract, say time 0, and can “vary freely”, thus affecting the time-0 price of the extendible call.)

Your job now is to come up with a formula that makes it possible to actually compute the (arbitrage-free) price of a specific extendible call. The “computation” could be done by simulation (OK), numerical solution of a PDE (good), numerical integration (even better), or from a a closed-form solution (excellent). (Hint: Look closely at what happens at \(T_1\) and recall what you know about prices of ordinary calls.)

You are not supposed to do any actual numerical calculations, but try to devise a price formula such that someone who knows programming & numerics, but nothing about finance, can find the price. And while we’re at it you may want to think about the following questions: What happens if \(A = 0\) ? What happens if \(K_1 \rightarrow \infty\) ? (This results in “an option on an option” aka, a **compound option**.) Is the option always extended? Never extended? What about hedging the option?

References


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<sup>Really, Fun ‘just kidding’. Don’t waste time trying to find a closed-form solution, Longstaff (1990) has done it and the result is a fairly complicated expression involving the bivariate normal distribution function. </sup>