Comments to the July 1998 edition of by Thomas Björk’s
Arbitrage Theory in Continuous Time (as of November 6, 2001)

Chapter 2

Wouldn’t it be better to refer to the $B$-process as a bank account, rather than as a bond?

In Section 2.1.3 (on contingent claims in the one-period binomial model) trading in the contingent claim is not formally defined, trading strategies are only allowed to contain the stock and the bond. On the other hand all arbitrage arguments can be carried out by buying or selling one unit of the contingent claim, so a formal extended definition of trading strategies might be overkill. The same comment applies to Section 2.2.2 (on contingent claims in the multiperiod binomial model).

On page 15 one could wonder why the subscripts in the equations giving bond and stock dynamics have suddenly been named $n$ (and not $t$ – but then one is being very petty).

In Definition 2.13 on page 16 where portfolio strategies are defined it says ‘we set $h_0 = h_1$ by convention’. I don’t see why ‘we’ want that, especially since the self-financing condition in Definition 2.14 on page 17 taken for $t = 0$ then reads

$$x_0(1 + R) + y_0S_0 = x_0 + y_0S_0,$$

implying that either $x_0$ (and hence $x_1$) equals 0, or that $R = 0$.

In Section 2.4, Stephen Ross has mysteriously disappeared form the reference to the 1979-paper on binomial option pricing (as it is the case throughout the rest of the book).

Chapter 3

In Definition 3.1 on page 27 it might be stressed that $N[a, b]$ denotes a normal distribution with with mean $a$ and standard deviation (not variance) $b$. Despite the notation possibly being aesthetically superior (e.g. $a$ and $b$ would have ‘same units’) and used consistently, I have seen several people confused by it.

In line 7 on page 31 ‘$T_n = b$’ → ‘$t_n = b$’.

In Equation (3.14) on page 32 it says

$$\int_a^bg(s)dW(s) \text{ is } \mathcal{F}_b^W \text{ - measurable.}$$
However, the term 'measurability' has not been introduced, defined or explained. To make (3.14) consistent with Definition 3.2 it one could write

\[
\int_a^b g(s) dW(s) \in \mathcal{F}_b^W.
\]

But using \( \in \) to describe a relation between random variables and \( \sigma \)-algebras is probably one that annoys 'purists' and could confuse 'semi-advanced' readers.

**In line 3 on page 34** (the definition of sub/super-martingales): \( s \geq t \rightarrow s \leq t \).

**On page 46** in the 3rd-to-last equation:

\[
\sum_{k=1}^d \delta_{ij} d\tilde{W}_i \rightarrow \sum_{i=1}^d \delta_{ij} d\tilde{W}_i
\]

**On page 46** in the next-to-last equation, \( 'dt' \)'s are missing.

**On page 48** in the displayed equation between (3.40) and (3.41) there is some \( 'dt' \)-inconsistency. Putting \( 'dt' \)'s on the first, third and fourth 'term' should fix it.

**Chapter 4**

Right after the start of **Section 4.1 on page 52** there is a stray \( 'c' \).

In connection with **Proposition 4.1 on page 52-3** (on the properties of solutions of SDEs) one might want to indicate to (or remind) the readers what a Markov-process is.

**In Proposition 4.3 on page 57**: \( \sigma \) should take values in \( M(n, d) \).

**In the proof of Proposition 4.3 on page 57**: 'by (4.17)' \( \rightarrow \) 'by (4.20)'. And wouldn't it be better to define a process \( X \) by (4.19) and then have people take a(n Ito) look at \( \exp(-At)X(t) \)?

There is sign convention discrepancy regarding \( r \) when **Proposition 4.8** is compared to **Proposition 4.6** and **Exercise 4.12** (but the results are correct).

**In 'bullet point 2' of Proposition 4.9 on page 62** there should be something about \( F(t, X_t) \) being a martingale, otherwise the statement does not make sense. Alternatively, the first half of 'bullet point 1' could be 'moved up' and made a common premise.
In Exercise 4.1 on page 65 it should say

\[ 'R_t = \int_0^t e^{-as} dW_s', \]

and \( f(y, z, r) = y + z \cdot r \). Further, there is a small notational inconsistency in the use of \( dW(s) \) (in (4.49)) and \( dW_s \) (everywhere else).

Chapter 5

On page 69 line 13-14 it should say 'is divided into periods of length \( \Delta t \').

On page 70 and onwards: Why not use standard matrix notation? For example (5.1) would then read

\[ 'V(t) = \ldots = h^s(t - \Delta t)S(t)', \]

but it would take a bit of work putting in all the transposition signs. (And further: In the equation immediately following (5.1), why has coordinate-number-indication suddenly changed to superscripts, when it's subscripts everywhere else? )

In Equation 5.9 on page 71 there's a 'dt' too much.

In Definition 5.3 on page 73 it's probably better if it says

\[ 'u_i(t) = \frac{h_i(t)S_i(t)'}{V^h(t)}'. \]

On page 74 the second displayed equation should say

\[ 'D_i(t) = \int_0^t \delta_i(s)ds' \]

(it looks like a blank space too much in the \( \TeX/\LaTeX \)-file.)

Chapter 6

In Definition 6.5 on page 80 the definition of an arbitrage is 'wrong', or at least only requiring such portfolios to be non-existent is is not sufficient (for example Proposition 6.6 is then wrong.) Of course, the analogue of Definition 2.15 should be substituted.

In Proposition 6.9 on page 87 there is an unintended blank space after 'Risk neutral valuation'.

3
On page 89 in the third-to-last displayed equation it should say

\[ 'A = \frac{se^{i\tau}}{\sqrt{2\pi}} \int_{\gamma_0}^{\infty} e^{s\sqrt{\tau^2 - \frac{1}{4}}} ds \ldots ' , \]

i.e. there’s a z missing in the book.

On page 92 line 17-18 it should read ’by using Proposition 6.12 an eqn (6.50)’.

On page 92 in the “Black-76” is should be ’F = F(t, T_1)’.

In Section 6.10 a mysterious ‘,’ has crept into the reference to the Barone-Adesi and Elliot-paper.

Chapter 9

On page 123 line 4 is it perhaps clearer if it say ’equations’, since there are n equations.

On page 129 the 6th displayed equation should read

\[ 'F_{in}(t, s) = F_{in}(t, s) = \sum_{j=1}^{n-1} \ldots ' . \]

Further, I think the sign should be reversed in the 7th displayed equation on the same page.

In Equation (9.25) on page 129 it should probably say

\[ 'D_{ij} = C_{ij} + C_{nn} - C_{in} - C_{nj} ' . \]

In the first displayed PDE on page 131 s_1^2 and and s_2^2 are missing in front of F_{11} and F_{22}, respectively.

Chapter 11

On page 159 line 30 and Proposition 11.6 on page 160 the notation is more consistent if ’F^{a0}’ (rather than ’F_0’) is used.

Chapter 12

On page 170 something is evidently wrong immediately after Proposition 12.3.

On page 172 in the proof of Proposition 12.5 it should read ’the relation X = \overline{B_f}/B’ in the 2nd line, and ’the relation S_f = \overline{S_f}/X’ in the 3rd.

On page 180 Exercises 12.4 and 12.5 are exactly the same.
Chapter 14

On page 201 in line 5 it should read ‘... and \( C^n \) are...’.
On pages 207 and 208 it should say

\[
\ldots (A^n H)(s, X^n_t) \ldots,
\]

in the last displayed equation on the former page and in the 3rd latter.

On page 208 it should say

\[
\ldots F^g(t, x) \ldots
\]

in the displayed equation immediately after Equation (14.24).

On page 210 in Equation (14.29) is should read

\[
\ldots A^n V(t, x) \ldots
\]

Chapter 15

On page 234 in the proof of Proposition 15.5 the 3rd-to-last displayed equation it should say

\[
\ldots - \int_t^T \sigma(t, s) dW(t) ds. \]

Maybe it would clarify things if it said ' (an analogue to) the Leibniz formula’ somewhere in the proof. (And the same goes for the proof of the Musiela parametrization in Chapter 18.)

Chapter 17

On page 261 in Section 17.4.2 right parentheses are missing in the first lines of the curly-brace-expression for \( A \) and \( B \).

On page 264 Equation 17.47 should read

\[
' = f^*_T(0, T) + \dot{g}(T) + a\{f^*(0, T) + g(T)\}'.
\]

Chapter 18

On page 274 in Exercise 18.5 the consol dynamics should be

\[
dC(t) = (C(t)r(t) - 1)dt + \sigma_C(t)dW(t).
\]
Chapter 19

On page 293 Equation 19.63 should read

\[ 'dr = [\Theta(t) - ar - \sigma^2 B(t, T)] dt + \sigma dW^T, \]

and in Equation 19.64, the first term on the RHS should be multiplied by \( r_t \), and a correction similar to the one in 19.63 should also be made.

In the Notes on page 297 it should read ‘Geman (1989)’ (which is also what it does in the bibliography).

References

The year for the Heath-Jarrow-Morton Econometrica article should be 1992.