Brownian motion (or not?)

In the following, $W = (W_t)_{t \geq 0}$ denotes a standard Brownian motion (SBM), typically 1-dimensional. Unless explicitly stated, when we say that something is a SBM we mean wrt. its own filtration.

Let $s \in \mathbb{R}_+$ and show that the process $(W_{s+t} - W_s)_{t \geq 0}$ is a SBM.

Let $s \in \mathbb{R}_+$ and show that the process $(W_{s+t} - W_s)_{t \leq s}$ is a SBM restricted to $[0, s]$ (which means what you think it means).

Let $a \in \mathbb{R}_+$ and show that the process $(\sqrt{a} W_{t/a})_{t \geq 0}$ is a SBM.

Show that the process defined by

$$X_t = \begin{cases} 
    t W_t & \text{for } t > 0 \\
    0 & \text{for } t = 0
\end{cases}$$

is a SBM. (It's a bit tricky showing continuity a 0. If you can just show that for $n \in \mathbb{N}$ we have $\lim_{n \to \infty} X_{1/n} = 0$ a.s. and see why we're not quite there yet, it'll suffice for this course. If you want a full proof you can use that $Z(n) = \max_{0 \leq s \leq t} |W(s+n) - W(n)|$ defines independent, identically distributed random variables that (by something called Doob's inequality) have finite mean. Because $Z(n) = \sum_{i=1}^{n} Z_i - \sum_{i=1}^{n-1} Z_i$, $Z(n)/n$ converges a.s. to 0, and the result follows by a little munging around. Do not waste time with this last part in class.)

Let $W = (W^{(1)}, W^{(2)}, \ldots, W^{(d)})^T$ be a (column) vector of independent SBMs and suppose $\alpha \in \mathbb{R}^d \setminus \{0\}$. Show that $\alpha^T W/|\alpha|$ defines a SBM.

Are the SBMs above also SBMs wrt. the filtration of $W$?

(Skip this question if you've never heard of a Poisson-process.) Let $N = (N(t))_{t \geq 0}$ be a Poisson-process with intensity $\lambda$, and define the process $M$ by $M_t = (N_t - \lambda t)/\sqrt{\lambda}$. Find the mean and covariance functions for $M$. Does $M$ have independent increments? Is $M$ a SBM?

Suppose $Y \sim N(0, 1)$ and we define the process $X$ by $X_t = \sqrt{t} Y$. What is the distribution of $X_t$? Does $X$ have continuous sample paths? Is $X$ a SBM?