Remarks on Sharpe’s Section 3.7

If a specific agent, \( k \), say the \( k \)th, buys \( kq \) units of a specific non-simple asset, say the \( i \)th, for a price of \( P_i^M \) units of time-0 consumption (think of this as one posted by the market maker; hence the \( M \)) then his or her expected utility becomes

\[
E(U) = u(kX_1 - kq \cdot P_i^M) + \sum_{j=2}^{S} \pi_j \cdot d_j \cdot u(kX_j + kq \cdot Z_{i,j}),
\]

(1)

where — in addition to “standard Sharpe notation” \( \pi \) for probabilities, \( d \) for discount factors and \( u \) for utility functions — we

- number the states (possible outcomes, scenarios) from 1 (“time-0”, “now”), 2 (“time-1”, “later”) up to \( S \) (also “time-1”).
- write \( kX_j \) for agent \( k \)’s wealth/consumption is state \( j \) with his “pre-trade” portfolio.
- write \( Z_{i,j} \) for asset \( i \)’s payment in state \( j \).

For a given price \( P_i^M \) the agent will chose the \( kq \) that maximizes the expected utility. By differentiating (using the chain rule) (1) and setting it equal to 0, we get the first-order condition for optimality, which can be rewritten as

\[
P_i^M = \sum_{j=2}^{S} Z_{i,j} \cdot \frac{\pi_j \cdot d_j \cdot m(kX_j + kq \cdot Z_{i,j})}{m(kX_1 - kq \cdot P_i^M)},
\]

(2)

where \( m = u' \) is Sharpe’s notation for marginal utility, e.g \( m(x) = x^{-b} \) in the constant relative risk-aversion case.

Equation (2) cannot be solved (for \( kq \)) explicitly/algebraically/in closed form, but because its right-hand side is a nice, decreasing function, solving it numerically is unproblematic. Sharpe effectively suggests (and APSIM uses) the method known as bisection. In the Case1ByHand-spreadsheet it is done with Excel’s “goal seek”-function. (Both are just ways of making successively better guesses.)

\footnote{We have dependence on agent, asset and state. Indices that “go in front” refer to agents. I have not put it on utility functions where the agent dependence is inherent.}